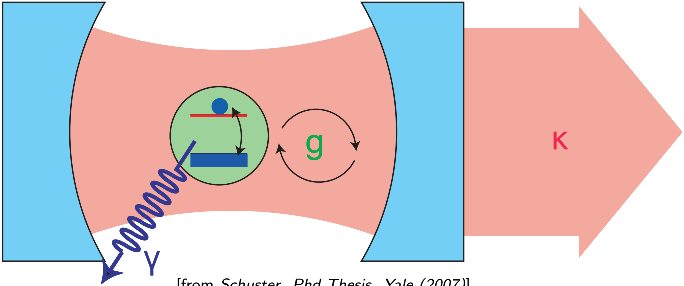
Introduction to Circuit QED

Michael Goerz

ARL Quantum Seminar
November 10, 2015

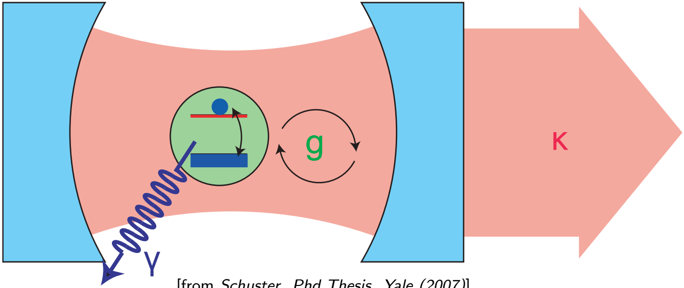
Jaynes-Cummings model



[from *Schuster. Phd Thesis. Yale (2007)*]

Jaynes-Cumming Hamiltonian

$$\hat{\mathbf{H}} = \frac{\omega_a}{2}\hat{\boldsymbol{\sigma}}_z + \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + g\left(\hat{\mathbf{a}}\hat{\boldsymbol{\sigma}}_+ + \hat{\mathbf{a}}^\dagger \hat{\boldsymbol{\sigma}}_-\right)$$



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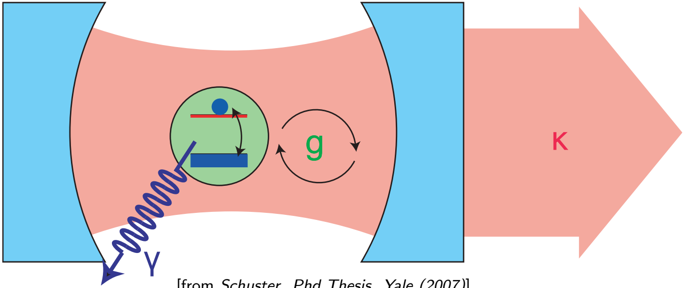
Outline

1. Superconducting qubits
2. Coplanar waveguide resonators
3. Combined system



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Outline

1. Superconducting qubits
2. Coplanar waveguide resonators
3. Combined system
4. Towards a network description of superconducting circuits

superconducting circuits and qubits

superconducting circuits

SC circuit toolbox: capacitors, inductors, Josephson elements

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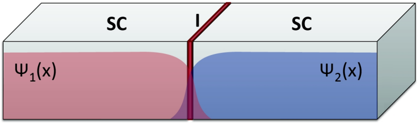
Josephson junction



[from *Shalibo. Phd Thesis. H. U. Jerusalem (2012)*]

superconducting circuits

SC circuit toolbox: capacitors, inductors, Josephson elements

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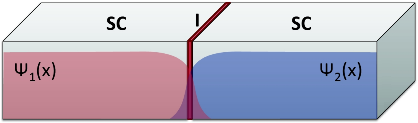

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- capacitance

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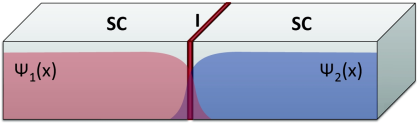

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- capacitance
- tunneling

$$I(t) = I_C \sin(\phi(t)); \; U(t) = \frac{\hbar}{2e}\frac{\partial \phi}{\partial t}$$

SC circuit toolbox: capacitors, inductors, Josephson elements

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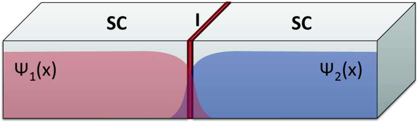

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$$\hat{\mathbf{H}} = \frac{(2e)^2}{2C_J}\left(\hat{\mathbf{n}} - \frac{Q_r}{2e}\right)^2 - \frac{\phi_0^2}{L_J}\cos\hat{\boldsymbol{\theta}}$$

SC circuit toolbox: capacitors, inductors, Josephson elements

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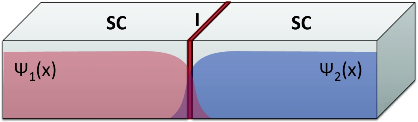

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cf. LC resonator

$$H = \frac{q^2}{2C} + \frac{\phi^2}{2L}$$

superconducting circuits

SC circuit toolbox: capacitors, inductors, Josephson elements

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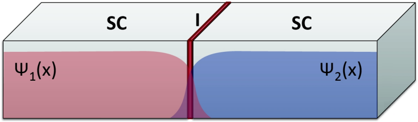

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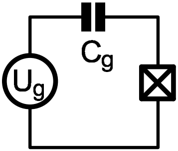
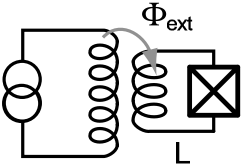
$$H = \frac{q^2}{2C} + \frac{\phi^2}{2L}$$

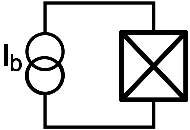
- superconductivity: macroscopic quantum coherence
- Josephson effect: *anharmonic* oscillator

types of superconducting qubits

standard SC qubits: charge qubit, flux qubit, phase qubit

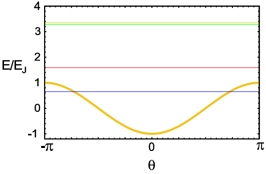


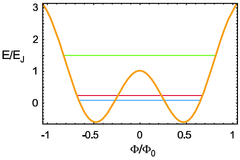


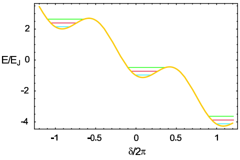


$E_C > E_J$ $E_J > E_C$ $E_J \gg E_C$

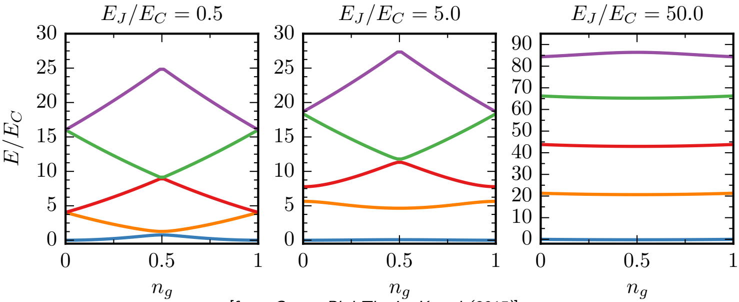
[from Devoret et al. arXiv:0411174 (2004)]







transmon qubit



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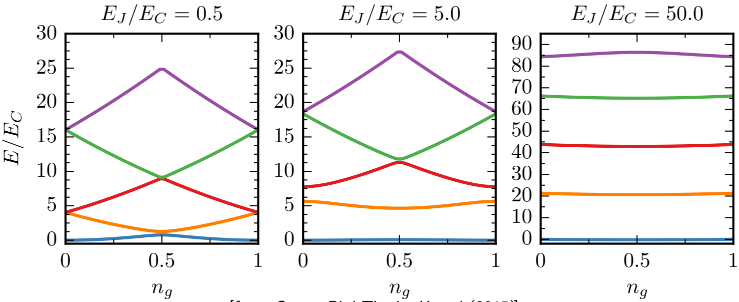

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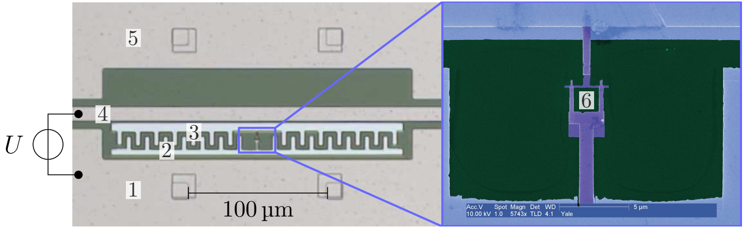

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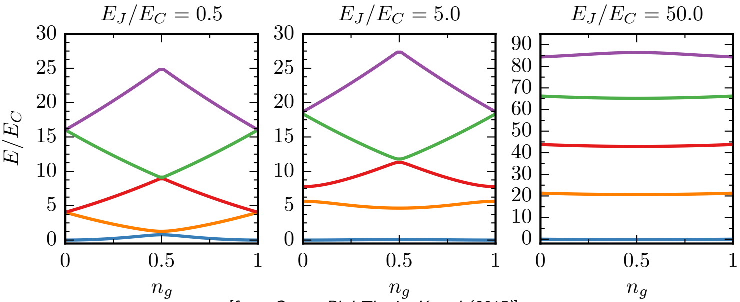

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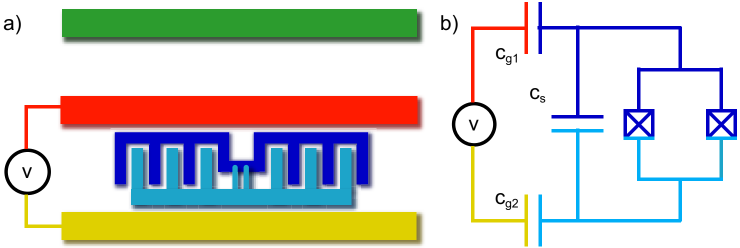

[from *Schuster. Phd Thesis. Yale (2007)*]

transmon Hamiltonian

Anharmonic Oscillator

$$\hat{\mathbf{H}} = 4E_C(\hat{\mathbf{n}} - n_g)^2 - E_J \cos\hat{\phi} \quad \text{for} E_J \gg E_C$$

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- Expand $\cos\hat{\phi}$ to $1 - \frac{\phi^2}{2} + \frac{\phi^4}{24}$, using HO $\hat{\mathbf{b}}^\dagger$, $\hat{\mathbf{b}}$ (Duffing Oscillator)

$$\hat{\mathbf{H}} = \sqrt{8E_C E_J}\,\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}} - \frac{E_C}{12}\left(\hat{\mathbf{b}}^\dagger + \hat{\mathbf{b}}\right)^4 + \text{const.}$$

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- Leading order perturbation theory on quartic term:

$$\hat{\mathbf{H}} = \omega_q\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}} + \frac{\alpha}{2}\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}}^\dagger\hat{\mathbf{b}}\hat{\mathbf{b}}$$

with $\omega_q \approx \sqrt{8E_J E_C}$, $\alpha \approx -E_c$.

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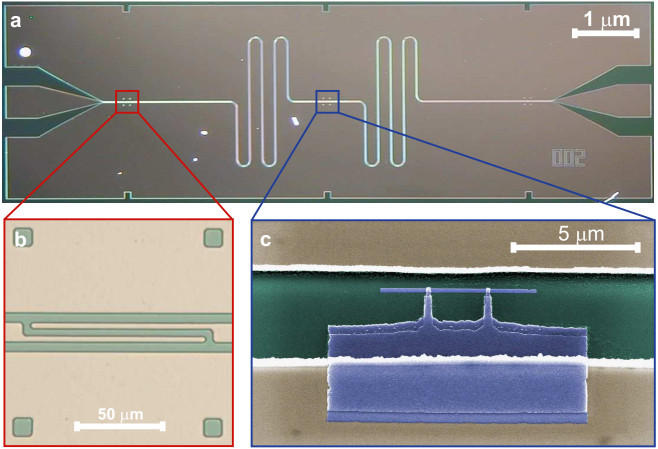
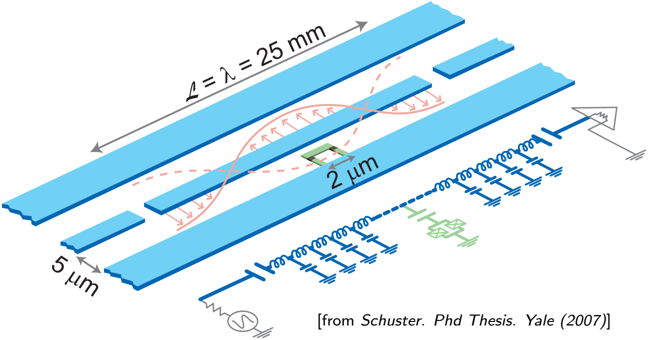
Example: $\alpha = -300$ MHz, $E_J/E_C = 50$, $\omega_q = 6$ GHz

Coplanar Waveguide Resonantors



[from *Schuster. Phd Thesis. Yale (2007)*]

distributed element description



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microwave pulses ⇒ lump element description inaccurate

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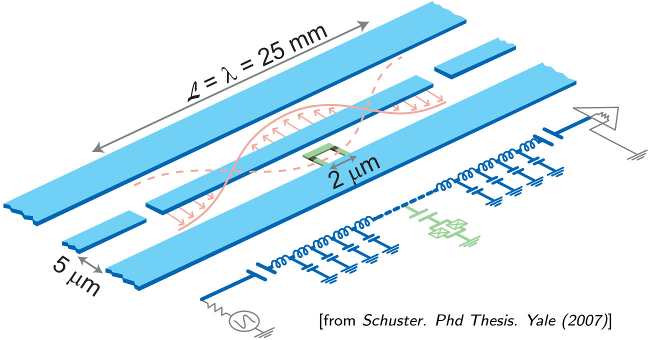

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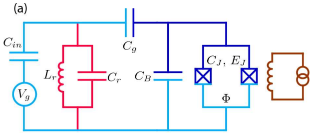
microwave pulses ⇒ lump element description inaccurate

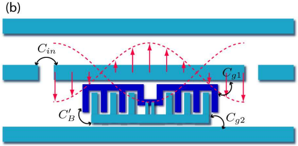
⇒ series of infinitessimal LC circuits

see Blais et al, PRA 69, 062320 (2004)

combined system

coupling the transmon to a cavity





from: J. Koch. PRA 76, 042319 (2007)

Jaynes-Cummings Hamiltonian

TLS in an optical cavity

$$\hat{\mathbf{H}} = \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \frac{\omega_q}{2} \hat{\boldsymbol{\sigma}}^+ \hat{\boldsymbol{\sigma}}^- + g \left(\hat{\mathbf{a}} + \hat{\mathbf{a}}^\dagger \right) \left(\hat{\boldsymbol{\sigma}}^- + \hat{\boldsymbol{\sigma}}^+ \right)$$

RWA if $\omega_c - \omega_q \ll \omega_c + \omega_q$:

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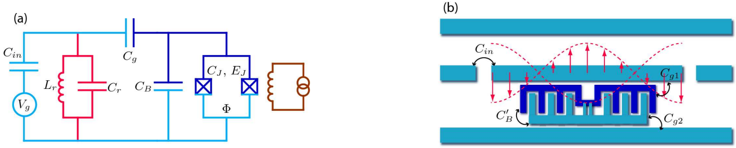

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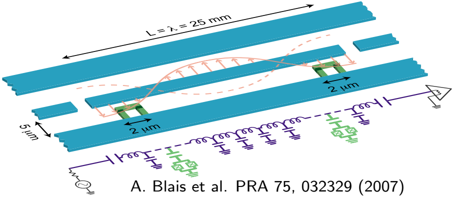


A. Blais et al. PRA 75, 032329 (2007)

Cavity mediates

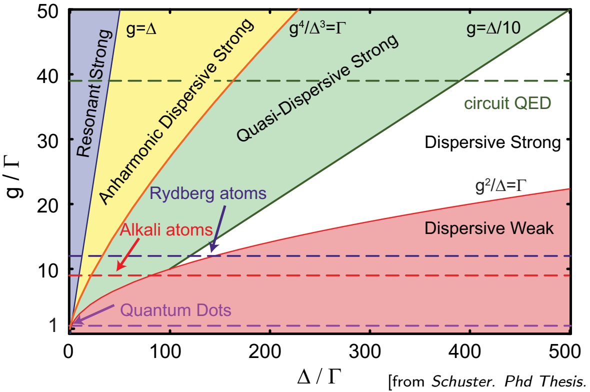
- driven excitation of qubit
- interaction between left and right qubit

Full Hamiltonian

$$\hat{\mathbf{H}} = \underbrace{\omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}_{①} + \underbrace{\omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2}_{②} - \underbrace{\frac{1}{2}(\alpha_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \alpha_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2)}_{③} +$$

$$+ \underbrace{g_1(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2(\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger)}_{④} + \underbrace{\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger}_{⑤}$$

parameter regimes

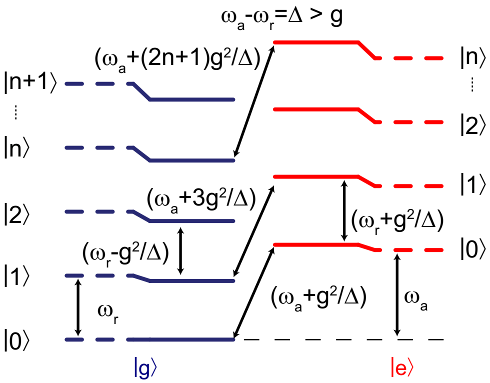


[from *Schuster. Phd Thesis. Yale (2007)*]

- dispersive:
  $g \ll \Delta$
- strong coupling:
  $g \gg \Gamma$

	3D optical	1D circuit
$\omega_r/2\pi$	350 THz	10 GHz
$g/2\pi, g/\omega_r$	220 MHz, 10^{-7}	100 MHz, 10^{-2}
$1/\kappa,\ Q = \frac{\omega_r}{\kappa}$	10 ns, 10^6	1 μs, 10^4
$1/\gamma$	50 ns	10 μs

dispersive frame



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dispersive shift:

$$\chi = \frac{g^2}{\Delta}$$

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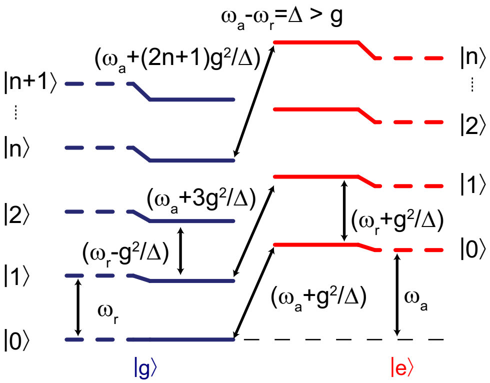

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dispersive shift:

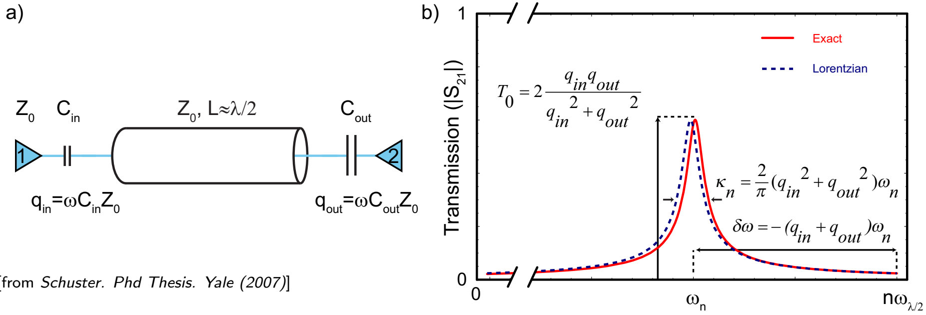
$$\chi = \frac{g^2}{\Delta}$$

dispersive frame: $\hat{\mathbf{U}} = \exp\left[\frac{g}{\Delta}\left(\hat{\mathbf{a}}\hat{\boldsymbol{\sigma}}_{+} - \hat{\mathbf{a}}^{\dagger}\hat{\boldsymbol{\sigma}}_{-}\right)\right]$

$$\Rightarrow \hat{\mathbf{U}}\hat{\mathbf{H}}\hat{\mathbf{U}}^{\dagger} = (\omega_r - \chi\hat{\boldsymbol{\sigma}}_z)\hat{\mathbf{a}}^{\dagger}\hat{\mathbf{a}} + \frac{1}{2}\left(\omega_a + \chi\right)\hat{\boldsymbol{\sigma}}_z$$

towards a network description of superconducting circuits

transmission properties of resonator



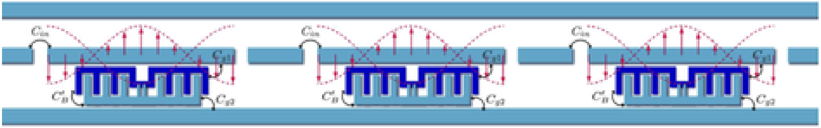
[from *Schuster. Phd Thesis. Yale (2007)*]

Impedance mismatch at capacitor acts as mirror

Input/Output behavior given by scattering matrix (transmission + reflection)

superconducting networks

- arrays of cavities?



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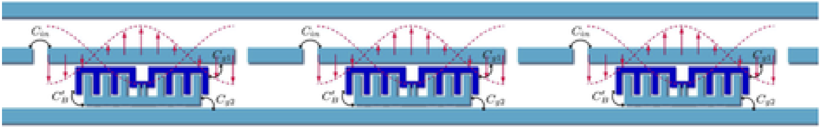

- beam splitters ⇒ capacitive junctions (back-reflection!)

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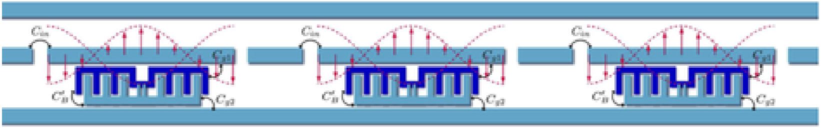

- beam splitters ⇒ capacitive junctions (back-reflection!)
- phase shifters?

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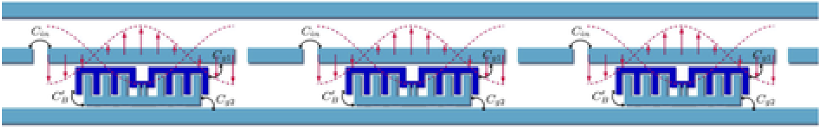

- beam splitters $\Rightarrow$ capacitive junctions (back-reflection!)
- phase shifters?
- T-junctions

superconducting networks

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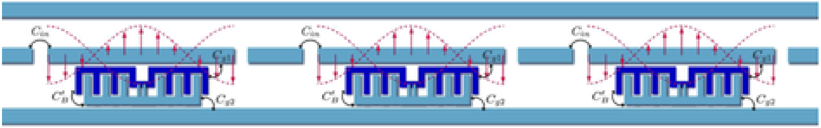

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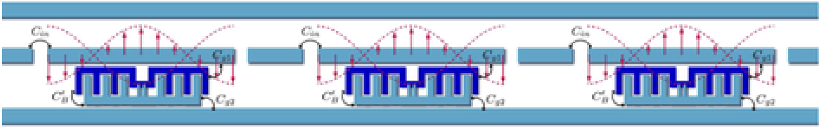

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- T-junctions
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⇒ electrical engineering methods for microwave engineering
Book: D. Pozar. Microwave Engineering 4th Ed. Wiley (2012).

summary & outlook

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- circuits yield $\hat{\mathbf{H}}$ in canonical variables charge, flux

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- transmission lines: distributed element description

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