Optimal Control Theory for Quantum Gates with Rydberg Atoms and Superconducting Qubits under Dissipative Dynamics

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UC Berkeley

Outline

Part I

- Optimal control theory for a unitary operation under dissipative evolution
  - Example 1: Controlled-Phase Gate with Rydberg Atoms
  - Example 2: $\sqrt{i\mathrm{SWAP}}$ using Transmon Qubits

Part II

- Optimizing a Rydberg Gate for Robustness
- Optimal Control of Superconducting Qubits

Part I

OCT for a unitary operation under dissipative evolution

D. Reich, G. Gualdi, C.P. Koch. PRA 88, 042309 (2013)
M. Goerz, D. Reich, C.P. Koch. arxiv:1312.0111

Standard approach to quantum gate optimization

$$\text{CPHASE} = \text{diag}(-1, 1, 1, 1)$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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Goal: Maximize

$$F = \frac{1}{d} \sum_{i=1}^{d} \Re\mathfrak{e} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{P}} \hat{\mathbf{U}}(T, 0, \epsilon) \hat{\mathbf{P}} \middle| \Psi_i \right\rangle$$

Two-qubit gates: $d = 4$

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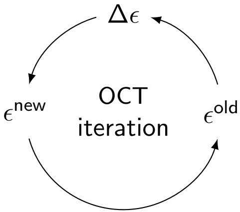
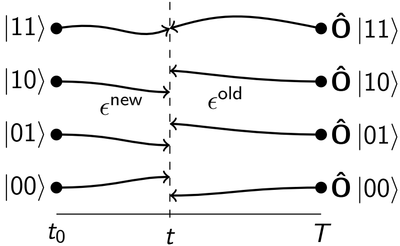


$$\Delta\epsilon(t) \propto \left\langle \chi(t) \middle| \partial_\epsilon \hat{\mathbf{H}} \middle| \Psi(t) \right\rangle$$



In the real world: decoherence

OCT for open quantum systems

$$\hat{\rho}(T) = \mathcal{D}(\hat{\rho}(0)); \qquad \text{e.g. } \frac{\partial \hat{\rho}}{\partial t} = \frac{i}{\hbar}[\hat{\mathbf{H}}, \hat{\rho}] + \mathcal{L}_D(\hat{\rho})$$

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Lift $F = \frac{1}{d} \sum_{i=1}^{d} \mathfrak{Re} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{P}} \hat{\mathbf{U}}(T, 0, \epsilon) \hat{\mathbf{P}} \middle| \Psi_i \right\rangle$ to Liouville space.

Kallush & Kosloff, Phys. Rev. A 73, 032324 (2006),
...
Schulte-Herbrüggen et al., J. Phys. B 44, 154013 (2011)

$$\Rightarrow F = \frac{1}{d^2} \mathfrak{Re} \sum_{j=1}^{d^2} \mathrm{tr}\left[\hat{\mathbf{O}} \hat{\rho}_j(0) \hat{\mathbf{O}}^\dagger \hat{\rho}_j(T)\right]$$

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$$\hat{\rho}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\rho}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\rho}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \ldots$$

d^2 matrices to propagate! (16 for two-qubit gate)

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Claim

We only need to propagate **three** matrices (independent of d), instead of d^2.

A reduced set of density matrices

No need to characterize the full dynamical map!

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① **Do we stay in the logical subspace?**

A reduced set of density matrices

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$$\hat{\rho}_3 = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\hat{\rho}_1 = \frac{1}{20}\begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \qquad \hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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E.g. $\hat{\mathbf{O}} = \text{diag}(-1, 1, 1, 1)$;
For $\hat{\mathbf{U}} = \mathbb{1}$
using just $\hat{\rho}_1$ will not distinguish $\hat{\mathbf{U}}$ from $\hat{\mathbf{O}}$. ($\hat{\mathbf{U}}\hat{\rho}_1\hat{\mathbf{U}}^\dagger = \hat{\mathbf{O}}\hat{\rho}_1\hat{\mathbf{O}}^\dagger$)

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$\hat{\rho}_1$, $\hat{\rho}_2$, $\hat{\rho}_3$ together guarantee that $\mathcal{D}(\hat{\rho})$ is unitary on the logical subspace.

A reduced set of density matrices

- $\hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

  dynamical map in the logical subspace

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  gate is diagonal in the same basis as $\hat{\mathbf{O}}$

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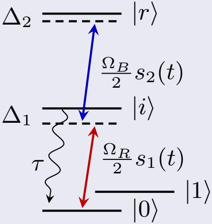
  Totally rotated state: relative phases between mapped logical eigenstates

- $\hat{\rho}_3 = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

  dynamical map in the logical subspace

Example 1: Optimization of a Rydberg Gate

Single-qubit Hamiltonian



$$\hat{\mathbf{H}}_{1q} = \begin{pmatrix} 0 & 0 & \frac{\Omega_R}{2} s_1(t) & 0 \\ 0 & E1 & 0 & 0 \\ \frac{\Omega_R}{2} s_1(t) & 0 & \Delta_1 & \frac{\Omega_B}{2} s_2(t) \\ 0 & & \frac{\Omega_B}{2} s_2(t) & \Delta_2 \end{pmatrix}$$

Two trapped neutral atoms

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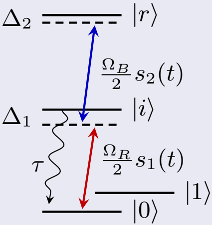

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Two-qubit Hamiltonian

$$\hat{\mathbf{H}}_{2q} = \hat{\mathbf{H}}_{1q} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{H}}_{1q} - U \left|rr\right\rangle \left\langle rr\right|$$

dipole-dipole interaction when both atoms in Rydberg state

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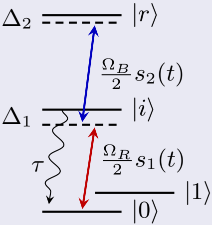

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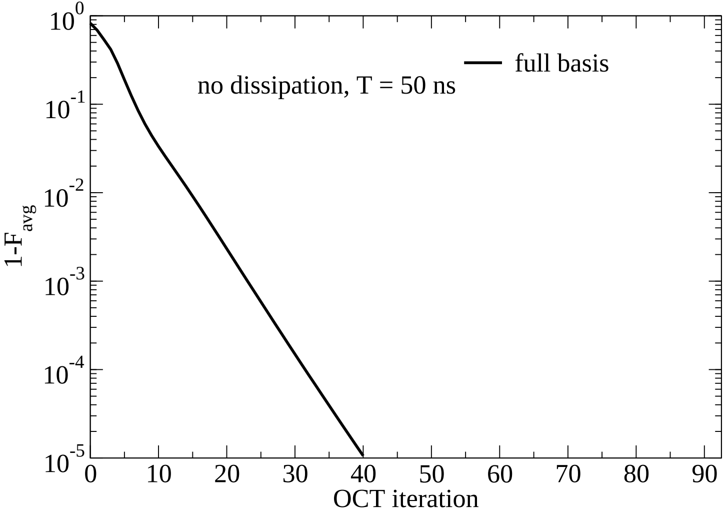
no coupling between $|0\rangle$, $|1\rangle$ $\Rightarrow$ only diagonal gates

$$\hat{\mathbf{U}} = \mathrm{diag}(e^{i\phi_{00}}, e^{i\phi_{01}}, e^{i\phi_{10}}, e^{i\phi_{11}})$$

first: optimize in Liouville space
– but without dissipation

OCT with a reduced set of states... without dissipation



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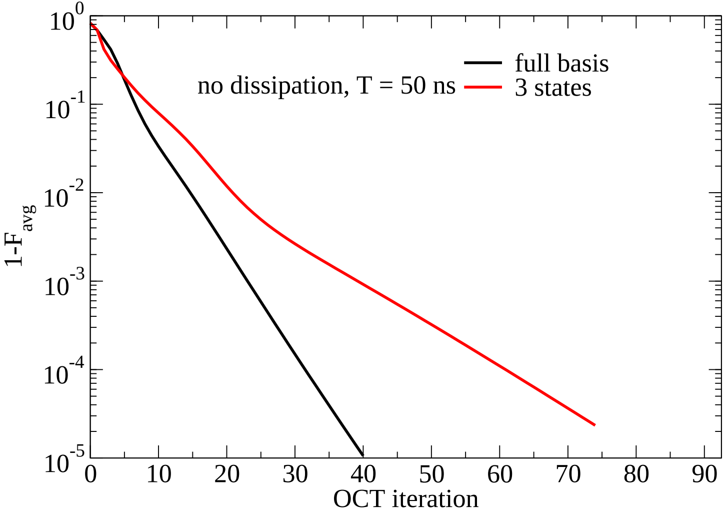
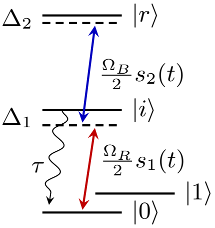


Diagonal Gates



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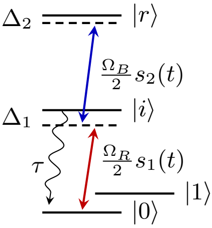

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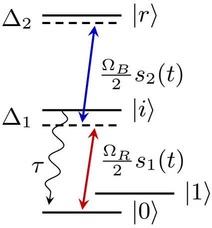

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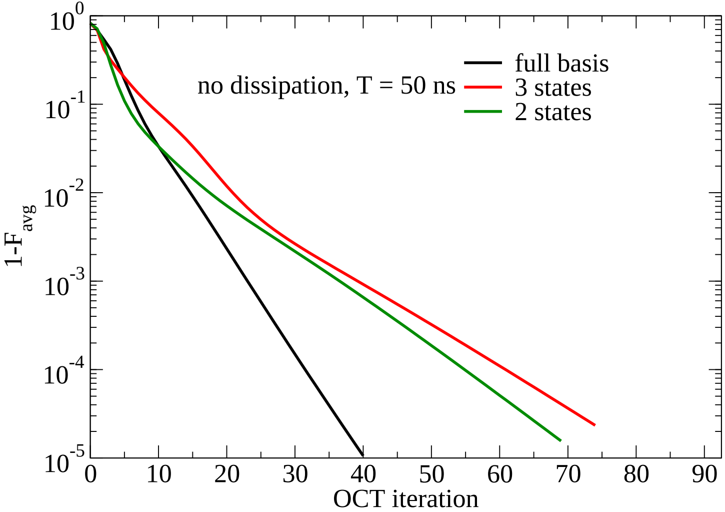
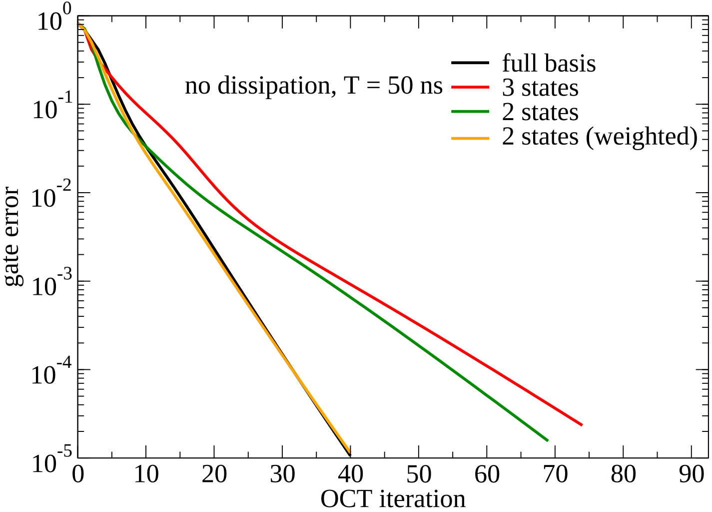
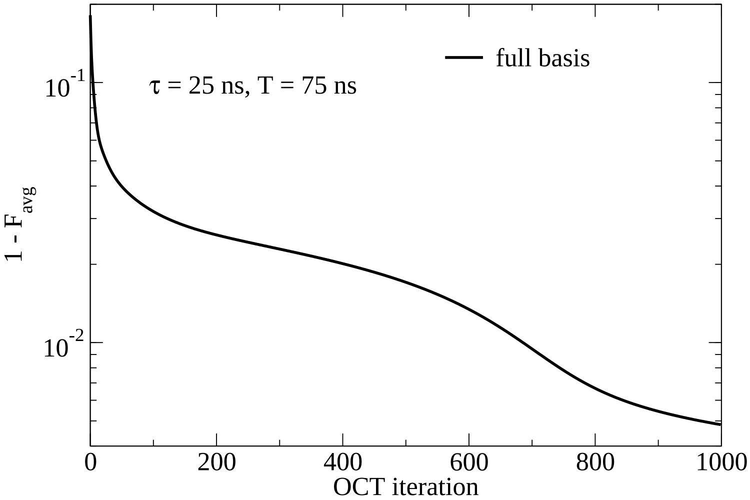
OCT with a reduced set of states...without dissipation

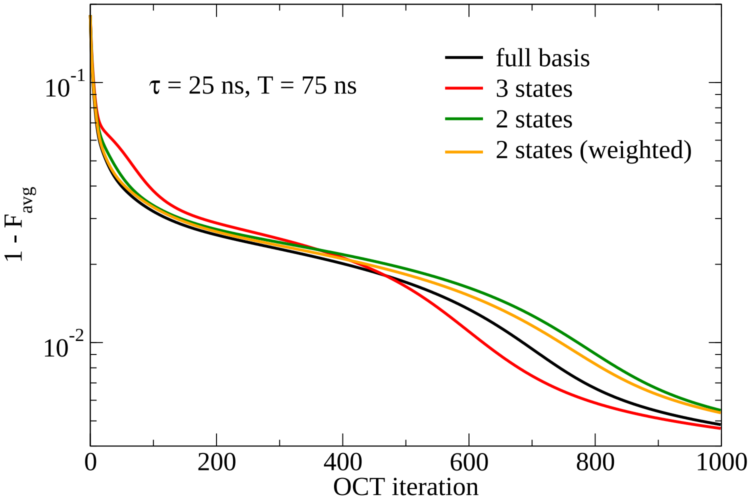


OCT with a reduced set of states. . . without dissipation

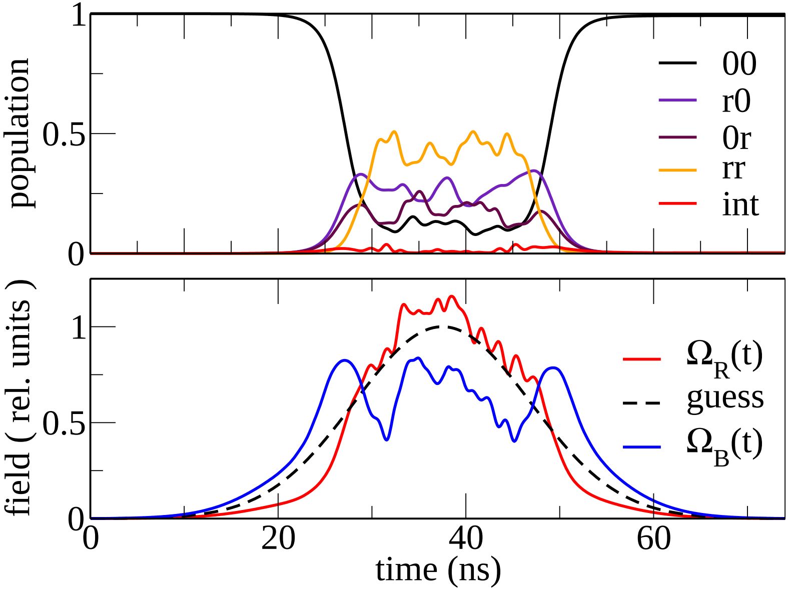


with dissipation


full basis
τ = 25 ns, T = 75 ns
10^-1
10^-2
1 - F_avg
0
200
400
600
800
1000
OCT iteration

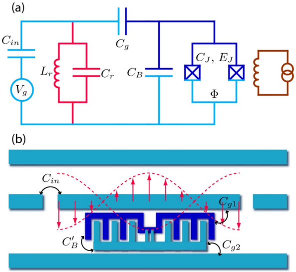

τ = 25 ns, T = 75 ns
full basis
3 states
2 states
2 states (weighted)
1 - F avg
10-1
10-2
0
200
400
600
800
1000
OCT iteration

Optimized dynamics

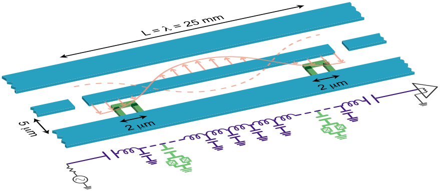


Example 2: Optimization of a Transmon Gate

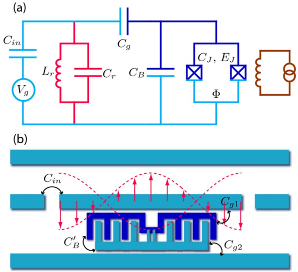
Two Coupled Transmon Qubits



J. Koch et al. PRA 76, 042319 (2007)



A. Blais et al. PRA 75, 032329 (2007)



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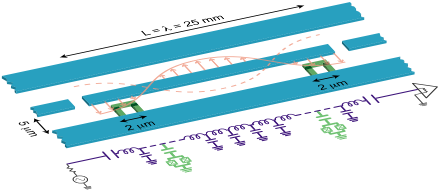

A. Blais et al. PRA 75, 032329 (2007)

Full Hamiltonian

$$\hat{\mathbf{H}} = \underbrace{\omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}_{①} + \underbrace{\omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2}_{②} - \underbrace{\frac{1}{2}(\alpha_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \alpha_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2)}_{③} +$$

$$+ \underbrace{g_1(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2(\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger)}_{④} + \underbrace{\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger}_{⑤}$$

Effective Hamiltonian

$$\hat{\mathsf{H}}_{\text{eff}} = \sum_{q=1,2} \sum_{i=0}^{N_q-1} (\omega_i^{(q)} + \chi_i^{(q)}) \hat{\mathsf{\Pi}}_i^{(q)} + \sum_{q=1,2} \sum_{i=0}^{N_q-1} g_i^{\text{eff}\,(q)} \epsilon(t) (\hat{\mathsf{C}}_i^{+\,(q)} + \hat{\mathsf{C}}_i^{-\,(q)})$$
$$+ \sum_{ij} J_{ij}^{\text{eff}} (\hat{\mathsf{C}}_i^{-\,(1)} \hat{\mathsf{C}}_j^{+\,(2)} + \hat{\mathsf{C}}_i^{+\,(1)} \hat{\mathsf{C}}_j^{-\,(2)}) \,.$$

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with

- $\omega_i^{(q)} = i\omega_q - \frac{1}{2}(i^2 - i)\alpha_q, \qquad g_i^{(q)} = \sqrt{i} g_q$
- $\hat{\mathbf{\Pi}}_i^{(q)} = \left|i\right\rangle\left\langle i\right|_q$, $\hat{\mathbf{C}}_i^{+\,(q)} = \left|i\right\rangle\left\langle i-1\right|_q$
- $\chi_i^{(q)} = \frac{(g_i^{(q)})^2}{(\omega_i^{(q)} - \omega_{i-1}^{(q)} - \omega_c)}$
- $g_i^{\text{eff}\,(q)} = \frac{g_i^{(q)}}{(\omega_i^{(q)} - \omega_{i-1}^{(q)} - \omega_c)}$
- $J_{ij}^{\text{eff}} = \frac{1}{2} g_i^{\text{eff}\,(1)} g_j^{(2)} + \frac{1}{2} g_j^{\text{eff}\,(2)} g_i^{(1)}$

qubit frequency ω_1	4.3796 GHz
qubit frequency ω_2	4.6137 GHz
drive frequency ω_d	4.4985 GHz
anharmonicity α_1	-239.3 MHz
anharmonicity α_2	-242.8 MHz
effective qubit-qubit coupling J	-2.3 MHz
qubit 1,2 decay time T_1	38.0 μs, 32.0 μs
qubit 1,2 dephasing time T_2^*	29.5 μs, 16.0 μs

Effective Hamiltonian

$$\hat{\mathbf{H}}_{\text{eff}} = \sum_{ijq} \left((\omega_i^{(q)} + \chi_i^{(q)}) \hat{\mathbf{\Pi}}_i^{(q)} + g_i^{\text{eff}\,(q)} \epsilon(t) (\hat{\mathbf{C}}_i^{+\,(q)} + \hat{\mathbf{C}}_i^{-\,(q)}) + J_{ij}^{\text{eff}} (\hat{\mathbf{C}}_i^{-\,(1)} \hat{\mathbf{C}}_j^{+\,(2)} + c.c.) \right)$$

Master Equation

$$\mathcal{L}_D(\hat{\boldsymbol{\rho}}) = \sum_{q=1,2} \left(\gamma_q \sum_{i=1}^{N-1} iD \left[|i-1\rangle\langle i|_q \right] \hat{\boldsymbol{\rho}} + \gamma_{\phi,q} \sum_{i=0}^{N-1} \sqrt{i} D \left[|i\rangle\langle i|_q \right] \hat{\boldsymbol{\rho}} \right) ,$$

with $D\left[\hat{\mathbf{A}}\right] \hat{\boldsymbol{\rho}} = \hat{\mathbf{A}} \hat{\boldsymbol{\rho}} \hat{\mathbf{A}}^\dagger - \frac{1}{2} \left(\hat{\mathbf{A}}^\dagger \hat{\mathbf{A}} \hat{\boldsymbol{\rho}} + \hat{\boldsymbol{\rho}} \hat{\mathbf{A}}^\dagger \hat{\mathbf{A}} \right)$

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- Near resonance of α_1 with $\omega_1 - \omega_2$

Effective Hamiltonian

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- Near resonance of α_1 with $\omega_1 - \omega_2$
- single frequency drive centered between two qubits

Effective Hamiltonian

$$\hat{\mathbf{H}}_{\text{eff}} = \sum_{ijq} \left((\omega_i^{(q)} + \chi_i^{(q)})\hat{\mathbf{\Pi}}_i^{(q)} + g_i^{\text{eff}\,(q)}\epsilon(t)(\hat{\mathbf{C}}_i^{+\,(q)} + \hat{\mathbf{C}}_i^{-\,(q)}) + J_{ij}^{\text{eff}}(\hat{\mathbf{C}}_i^{-\,(1)}\hat{\mathbf{C}}_j^{+\,(2)} + c.c.) \right)$$

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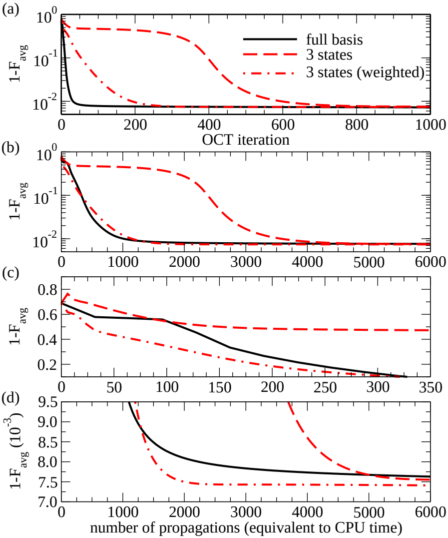
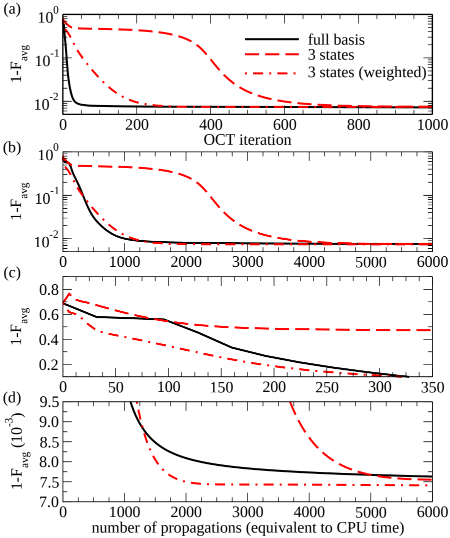
OCT with a reduced set of states

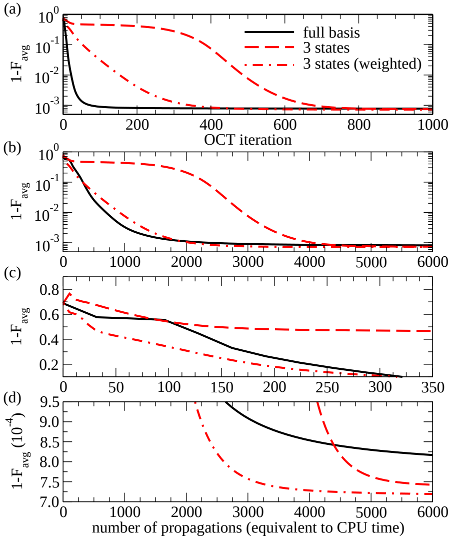


full dissipation

OCT with a reduced set of states

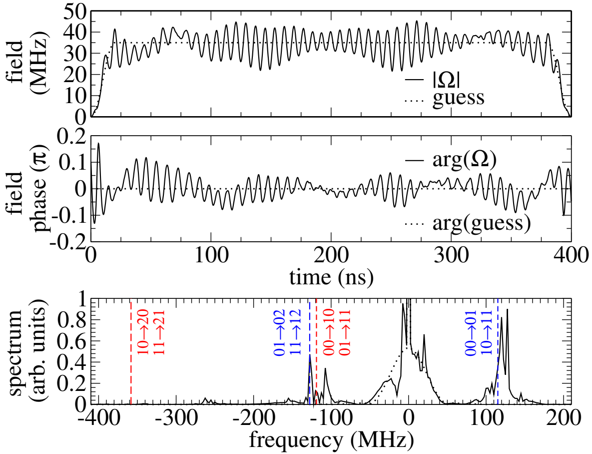


full dissipation

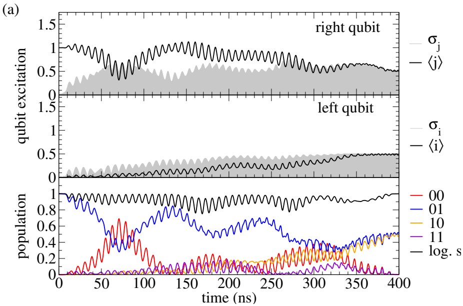


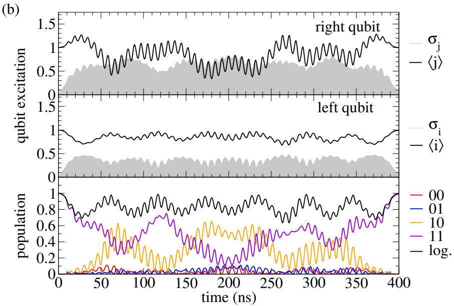
weak dissipation

Optimized Pulse



Population Dynamics





$\Psi(t = 0) = |01\rangle$ $\qquad$ $\Psi(t = 0) = |11\rangle$

Part II

Ongoing Projects

Part II

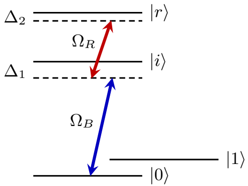
Ongoing Projects

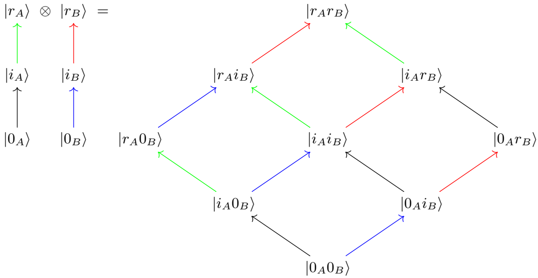
- Optimizing a Rydberg Gate for Robustness
- OCT for Superconducting Qubits

Optimizing a Rydberg Gate for Robustness

M. Goerz, E. Halperin, J. Aytac, C.P. Koch, K.B. Whaley.
Robustness of high-fidelity Rydberg gates with single-site addressability. In preparation.

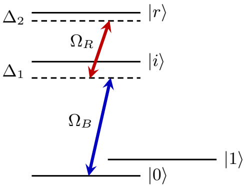
Jaksch-Zoller Scheme

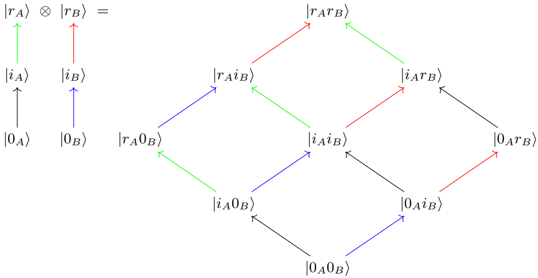




- blockade regime ($|rr\rangle$ blocked)
- single-site addressability (4 pulses)

Jaksch-Zoller Scheme





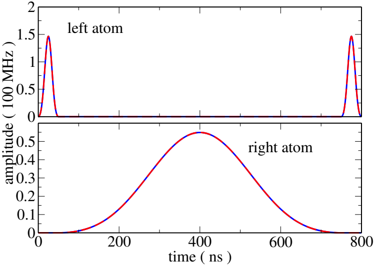
- blockade regime ($|rr\rangle$ blocked)
- single-site addressability (4 pulses)

Analytical pulse scheme: Jaksch et al. PRL 85, 2208 (2000)

	π-flip (l)		2π-flip (r)		π-flip (l)	
$\|00\rangle$	$\rightarrow$	$i\,\|r0\rangle$	$\rightarrow$	$i\,\|r0\rangle$	$\rightarrow$	$-\|00\rangle$
$\|10\rangle$	$\rightarrow$	$\|10\rangle$	$\rightarrow$	$-\|10\rangle$	$\rightarrow$	$-\|10\rangle$
$\|01\rangle$	$\rightarrow$	$i\,\|r1\rangle$	$\rightarrow$	$i\,\|r1\rangle$	$\rightarrow$	$-\|01\rangle$
$\|11\rangle$	$\rightarrow$	$\|11\rangle$	$\rightarrow$	$\|11\rangle$	$\rightarrow$	$\|11\rangle$

3-Level Transfer

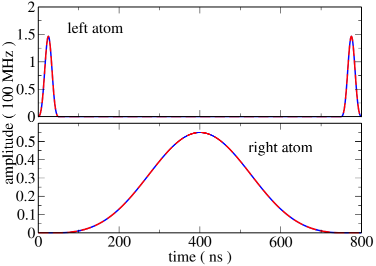
- Simultaneous pulses:



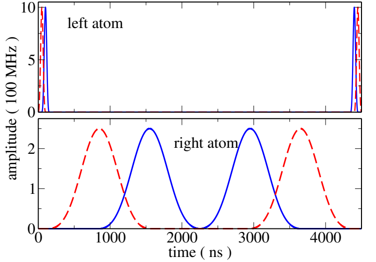
Problems:

- Simultaneous pulses: short (strong) pulses break blockage; population in $|i\rangle$

- Simultaneous pulses:



- STIRAP:

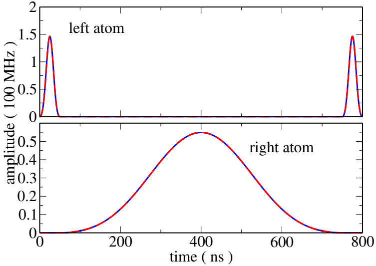


Problems:

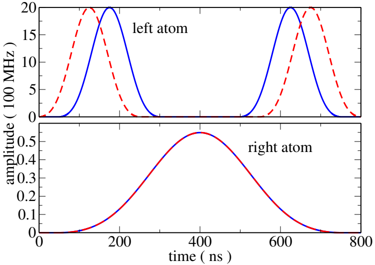
- Simultaneous pulses: short (strong) pulses break blockage; population in $|i\rangle$
- STIRAP: adiabaticity (slow); phase alignment is difficult

3-Level Transfer

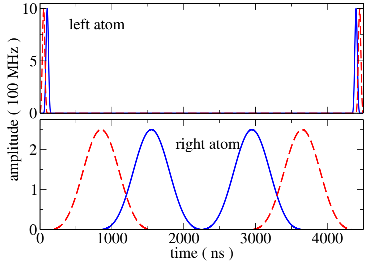
- Simultaneous pulses:



- Mixed:



- STIRAP:



Problems:

- Simultaneous pulses: short (strong) pulses break blockage; population in $|i\rangle$
- STIRAP: adiabaticity (slow); phase alignment is difficult

Mixed scheme: STIRAP is fine for π-pulses, just not for the 2π pulse

Robustness of Analytical Schemes

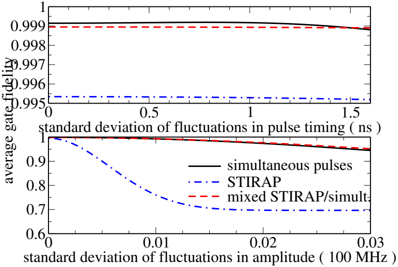


FIG. 9: Robustness of the Rydberg gate with respect to pulse timing inaccuracies (top) and amplitude fluctuations (bottom). All fluctuations are assumed to be Gaussian distributed.

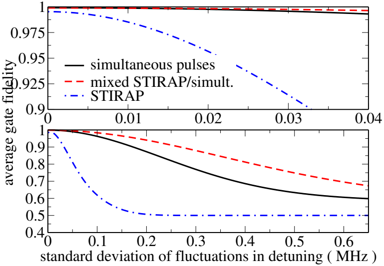


FIG. 10: Robustness of the Rydberg gate with respect to two-photon detuning for small detuning (top) and large detuning (bottom). All fluctuations are again assumed to be Gaussian distributed.

Optimizing for Robustness

Optimizing of an Ensemble of Hamiltonians

- fluctuations in pulse amplitude → fluctuations in dipole
- fluctuations in Rydberg level (external fields)

$$\Delta\epsilon(t) \propto \sum_{i=1}^{n} \left\langle \chi_i(t) \middle| \partial_\epsilon \hat{\mathbf{H}} \middle| \Psi_i(t) \right\rangle$$

Optimizing for Robustness

Optimizing of an Ensemble of Hamiltonians

- fluctuations in pulse amplitude $\rightarrow$ fluctuations in dipole
- fluctuations in Rydberg level (external fields)

$\Rightarrow$ $\hat{\mathbf{H}} \rightarrow$ ensemble $\{\hat{\mathbf{H}}_e\}$

$$\Delta\epsilon(t) \propto \sum_{e=1}^{N} \sum_{i=1}^{n} \left\langle \chi_{i,e}(t) \middle| \partial_\epsilon \hat{\mathbf{H}}_e \middle| \Psi_{i,e}(t) \right\rangle$$

Optimizing of an Ensemble of Hamiltonians

- fluctuations in pulse amplitude → fluctuations in dipole
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⇒ $\hat{\mathbf{H}} \to$ ensemble $\{\hat{\mathbf{H}}_e\}$

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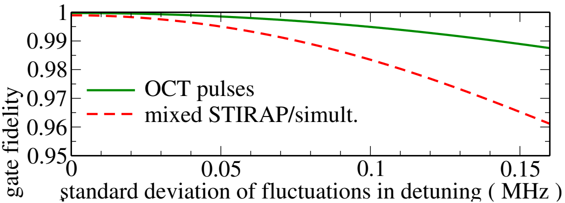
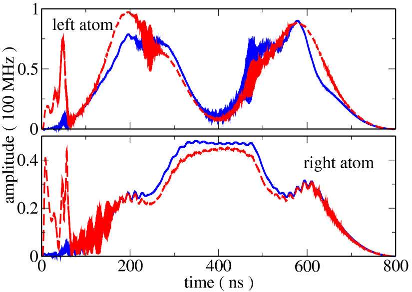
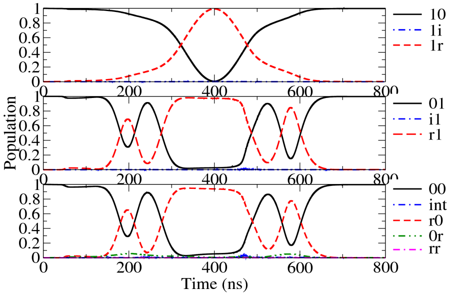


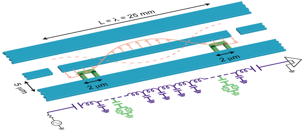


optimized pulses



population dynamics

OCT for Superconducting Qubits



A. Blais et al. PRA 75, 032329 (2007)

Full Hamiltonian

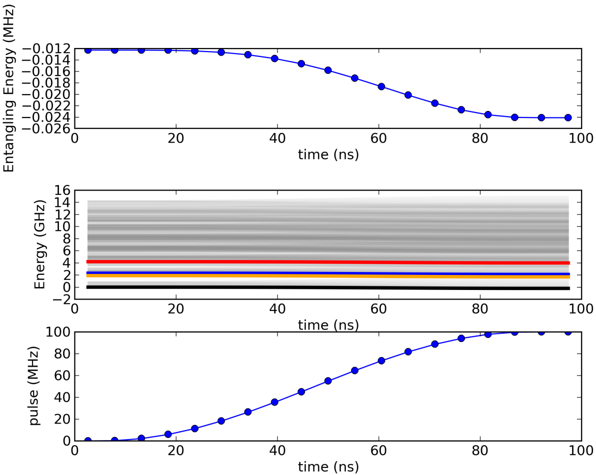
$$\hat{\mathbf{H}} = \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 - \frac{1}{2}\left(\alpha_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \alpha_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2\right)$$
$$+ g_1(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2(\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger) + \epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger$$

Effective Hamiltonian

$$\hat{\mathbf{H}}_{\text{eff}} = \sum_{q=1,2} \sum_{i=0}^{N_q-1} (\omega_i^{(q)} + \chi_i^{(q)}) \hat{\mathbf{\Pi}}_i^{(q)} + \sum_{q=1,2} \sum_{i=0}^{N_q-1} g_i^{\text{eff}\,(q)} \epsilon(t) (\hat{\mathbf{C}}_i^{+\,(q)} + \hat{\mathbf{C}}_i^{-\,(q)})$$
$$+ \sum_{ij} J_{ij}^{\text{eff}} (\hat{\mathbf{C}}_i^{-\,(1)} \hat{\mathbf{C}}_j^{+\,(2)} + \hat{\mathbf{C}}_i^{+\,(1)} \hat{\mathbf{C}}_j^{-\,(2)}) \,.$$

Dynamic Stark Shift on Qubit Levels

- Possible gate mechanism: Non-linear Stark shift on logical levels
- Interaction Energy $E_{00} - E_{10} - E_{01} + E_{11}$



Summary and Outlook

Efficient optimization of gates in open quantum systems:

- A set of three density matrices is sufficient for gate optimization: (independent of dimension of Hilbert space!)
  - one to check dynamical map on subspace
  - one to check the basis
  - one to check the phases
- Further reduction possible for restricted systems
- States can be weighted according to physical interpretation

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Efficient optimization of gates in open quantum systems:

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  - one to check the basis
  - one to check the phases
- Further reduction possible for restricted systems
- States can be weighted according to physical interpretation

Ongoing Projects:

- Optimizing for robustness is possible by optimizing over an ensemble of Hamiltonians
- Superconducting Qubits: Gate Mechanism…
  Controlled-Phase gates through non-linear Start shifts?

Thank You!