# Optimal Control for Quantum Networks

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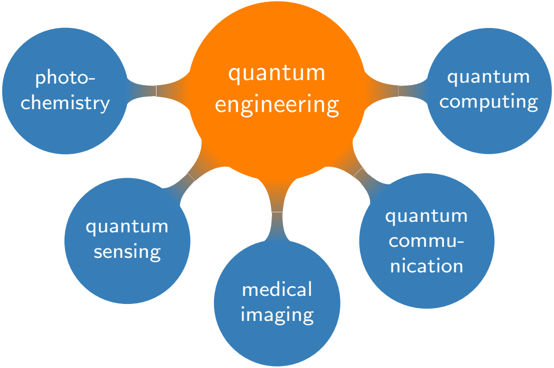
CECAM Workshop

Numerical methods for optimal control of open quantum systems

Berlin

September 27, 2016

# quantum technology and quantum networks



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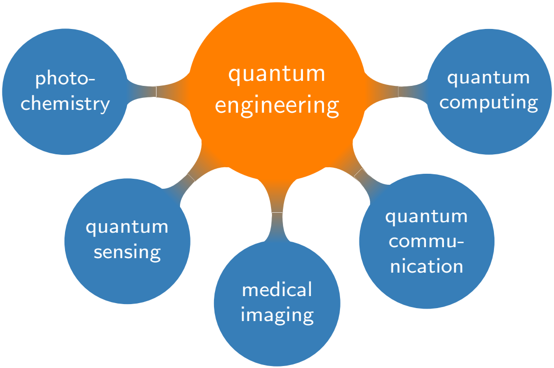



scalable systems ⇒ quantum networks

# the software toolbox

## Modeling

### QNET (python)

Design and analysis of photonic circuit models

- QHDL model
- SLH formalism
- symbolic quantum algebra
- circuit component library
- visualization

yields Master equation of quantum network

https://github.com/mabuchilab/qnet

## Simulation & Optimization

### QDYN (Fortran)

high performance quantum simulation and optimal control

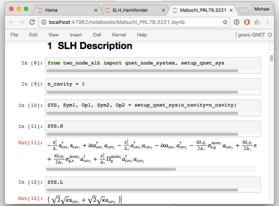
- Spectral methods
- Chebychev/Newton propagator
- Krotov's method
- Grape/LBFGS

Solves equation of motion and control problems

https://github.com/goerz/qdynpylib
http://bitly.com/agkoch-kassel

## Python Ecosystem

- jupyter
- sympy
- matplotlib




### QSD (C++)

Quantum Trajectories solver

https://github.com/mabuchilab/qsd-mpi

### clusterjob (python)

Drive HPC compute jobs

https://github.com/goerz/clusterjob

- QuTip

# numerical optimal control

## optimization functional

$$J_T = 1 - \frac{1}{d^2} \left| \sum_{k=1}^{d} \left\langle \phi_k^{\text{tgt}} \,\middle|\, \phi_k(T) \right\rangle \right|^2 \longrightarrow 0$$

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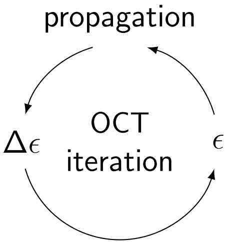
**iterative scheme:** $\epsilon^{(0)}(t) \rightarrow \epsilon^{(1)}(t)$



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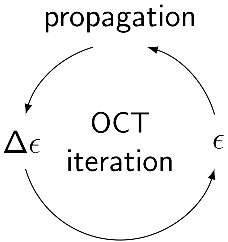
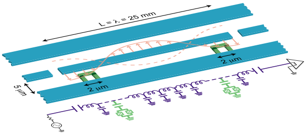



Applications:

- state preparation
- quantum gates, entanglement creation
- robustness to qu. and classical noise
- performance bounds (QSL, *parameter exploration*)

# mapping the design parameter landscape of cQED




[Blais et al, PRA 75, 032329 (2007)]

transmon qubits: optimal system parameters?

- qubit frequency, anharmonicity
- qubit-cavity coupling, detuning

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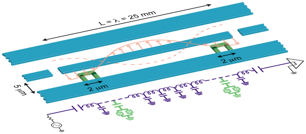


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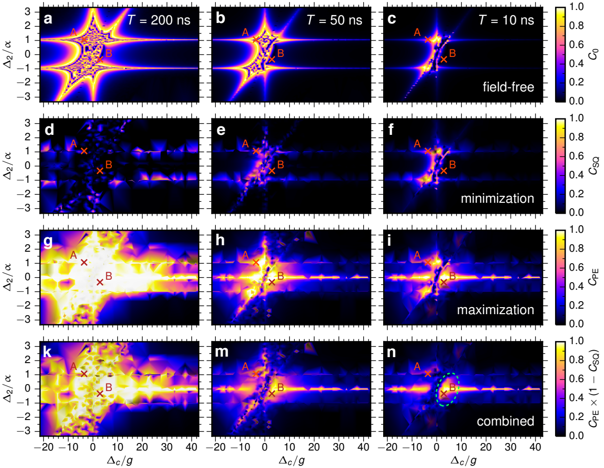
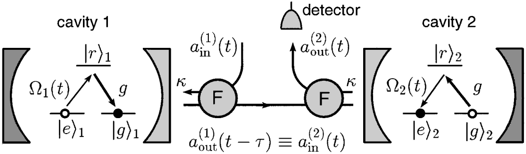



identify new parameter regime!

arXiv:1606.08825 (2016)

# quantum networks

# a two-node network

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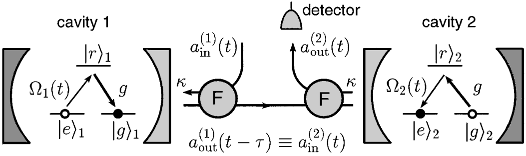


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- each node $j$ (after adiabatic elimination):
  - $\hat{\mathbf{H}}_j = -\delta \hat{\mathbf{a}}_j^\dagger \hat{\mathbf{a}}_j - i g_j(t)(\hat{\boldsymbol{\sigma}}_+ \hat{\mathbf{a}}_j - \hat{\boldsymbol{\sigma}}_- \hat{\mathbf{a}}_j^\dagger)$
  - Lindblad operator $\sqrt{2\kappa}\hat{\mathbf{a}}_j$

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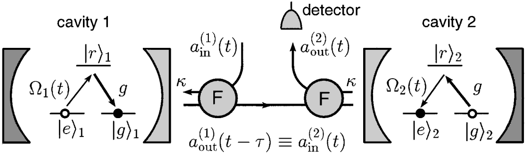


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- input-output theory (SLH framework): [Gough, James]
  - $\hat{\mathbf{H}} = \hat{\mathbf{H}}_1 + \hat{\mathbf{H}}_2 + i\kappa(\hat{\mathbf{a}}_1^\dagger \hat{\mathbf{a}}_2 - \hat{\mathbf{a}}_1 \hat{\mathbf{a}}_2^\dagger)$
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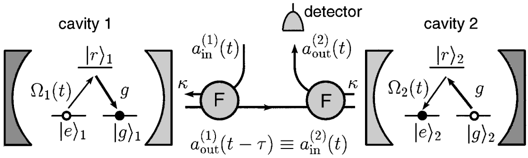


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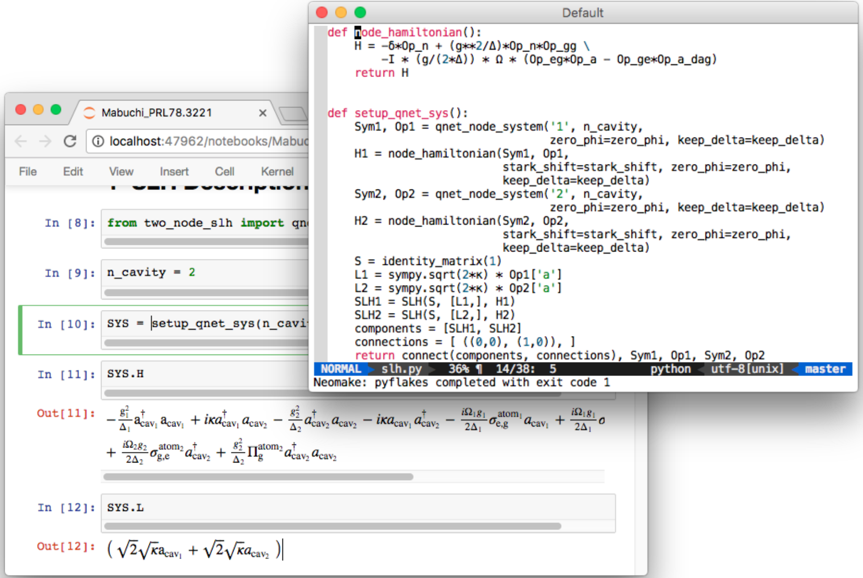
challenges:

- large combined Hilbert spaces (for larger networks)
- inherently dissipative (at the same scale as interactions!)

# a two-node network



# quantum trajectories

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**ensemble dynamics**

$$\hat{\rho}(t) = \frac{1}{N} \sum_{n=1}^{N\to\infty} \left|\Psi_n(t)\right\rangle \left\langle\Psi_n(t)\right|$$

$$\left\langle \hat{\mathbf{O}}(t) \right\rangle = \mathrm{tr}\left[\rho^\dagger \hat{\mathbf{O}}(t)\right] = \frac{1}{N} \sum_{n=1}^{N\to\infty} \left\langle \hat{\mathbf{O}}(t) \right\rangle_n$$

# the quantum jump (MCWF) method

for each trajectory $|\Psi_n\rangle$:

[Dum et al. PRA 4879 (1992); Mølmer et al. JOSAB 10, 524 (1993)]

1. effective Hamiltonian $H_{\text{eff}} = \hat{\mathbf{H}} - \frac{i\hbar}{2}\sum_i \hat{\mathbf{L}}_i^\dagger \hat{\mathbf{L}}_i$

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3. Apply an instantaneous quantum jump $|\Psi(t_j)\rangle \rightarrow \hat{\mathbf{L}}_n |\Psi(t_j)\rangle$ use $\hat{\mathbf{L}}_n$ with relative probability $\langle \Psi(t_j) | \hat{\mathbf{L}}_n^\dagger \hat{\mathbf{L}}_n | \Psi(t_j) \rangle$.

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Can we optimize over individual trajectories $|\Psi_n\rangle$?

# optimal control of quantum trajectories

# methods of optimal control – **gradient-free**

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- use e.g. Nelder-Mead simplex

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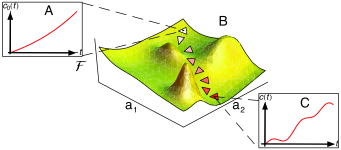


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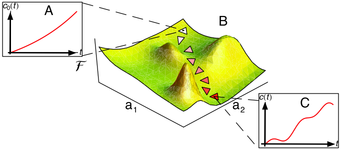


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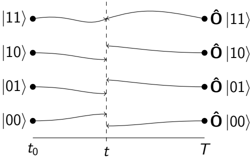
Works great when there are only a handful of control parameters.

Good for obtaining guess pulses!

typical functional: $J_T(\{\tau_k\})$,

$$\tau_k = \left\langle k^{\text{tgt}} \middle| \hat{\mathbf{U}}(T,0) \middle| k \right\rangle$$




- Grape/LBFGS: use gradient $\frac{\partial J_T}{\partial \epsilon_j}$

[Khaneja et al, JMR 172, 296 (2005); de Fouquiéres et al, JMR 212, 412 (2011)]

$$\frac{\partial \tau_k}{\partial \epsilon_j} = \left\langle k^{\text{tgt}} \middle| \hat{\mathbf{U}}_{nt-1} \dots \hat{\mathbf{U}}_{j+1} \frac{\partial \hat{\mathbf{U}}_j}{\partial \epsilon_j} \hat{\mathbf{U}}_{j-1} \dots \hat{\mathbf{U}}_1 \middle| k \right\rangle = \left\langle \chi_k(t_{j+1}) \middle| \frac{\partial \hat{\mathbf{U}}_j}{\partial \epsilon_j} \middle| \phi_k(t_j) \right\rangle ,$$

# methods of optimal control – **gradient-based**

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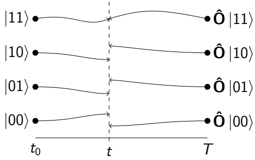


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- Krotov's method: constructive pulse update (time-continuous)

$$\Delta\epsilon(t) \propto \sum_{k=1}^{N} \left\langle \chi_k^{(i)}(t) \middle| \left( \frac{\partial \hat{\mathbf{H}}}{\partial \epsilon} \bigg|_{\substack{\phi^{(i+1)}(t) \\ \epsilon^{(i+1)}(t)}} \right) \middle| \phi_k^{(i+1)}(t) \right\rangle ; \quad \left|\chi_k^{(i)}(T)\right\rangle = - \frac{\partial J_T}{\partial \left\langle \phi_k \right|} \bigg|_{\phi_k^{(i)}(T)}$$

[Zhu et al, JCP 108, 1953 (1998); Palao, Kosloff, PRA 68 062308 (2003); Reich et al, JCP 136, 104103 (2012)]

# gradient-based trajectory optimization

**Grape/LBFGS:** $\frac{\partial \hat{\mathbf{U}}_j}{\partial \epsilon_j} \rightarrow \ldots ?$

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Each trajectory contributes to pulse update $\Delta\epsilon(t) \rightarrow$ average

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cf. "ensemble optimization" for robustness
[Goerz et al., PRA 90, 032329 (2014)]

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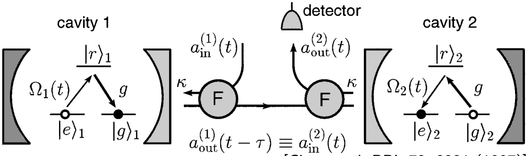
$$J_{T,sm} = \frac{1}{N^2}\left|\sum_k \tau_k\right|^2 \rightarrow -\left.\frac{\partial J_{T,sm}}{\partial \langle \phi_k |}\right|_{\phi_k^{(i)}(T)} = \left(\frac{1}{N^2}\sum_{l=1}^{N} \tau_l\right)\left|k^{\text{tgt}}\right\rangle,$$

$$J_{T,re} = \frac{1}{N}\mathfrak{Re}\sum_k \tau_k \rightarrow \quad -\left.\frac{\partial J_{T,re}}{\partial \langle \phi_k |}\right|_{\phi_k^{(i)}(T)} = \frac{1}{2N}\left|k^{\text{tgt}}\right\rangle$$

# example: directional state transfer

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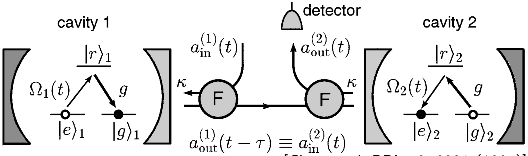


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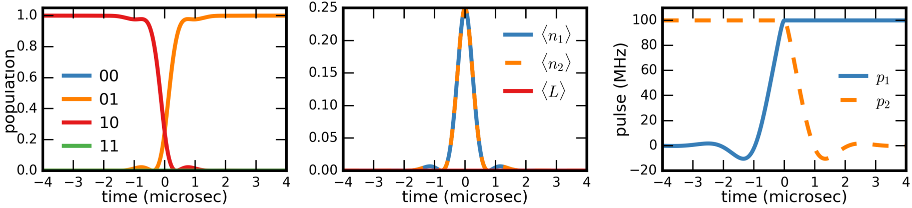
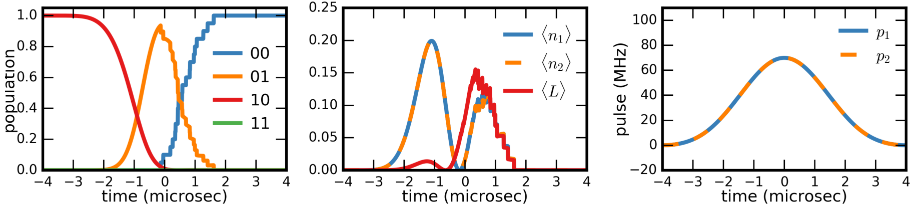
Time-symmetric solution to $|10\rangle \rightarrow |01\rangle$
with dark state condition $\hat{\mathbf{L}}\,|\Psi(t)\rangle = 0$



# optimal control solution for state transfer

**density matrix optimization:** $|10\rangle\langle 10| \rightarrow |01\rangle\langle 01|$ [Y. Ohtsuki]



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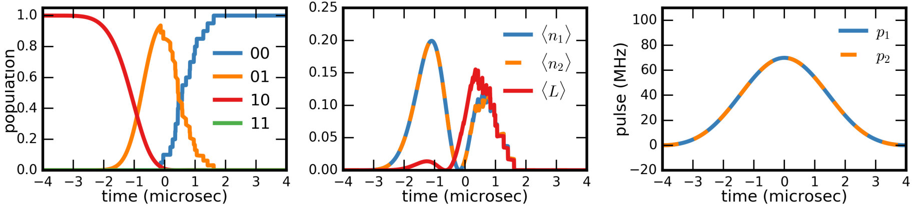

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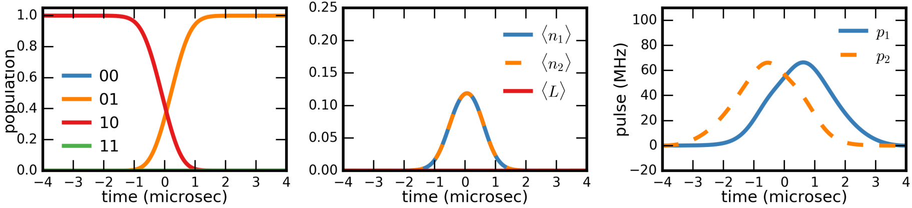


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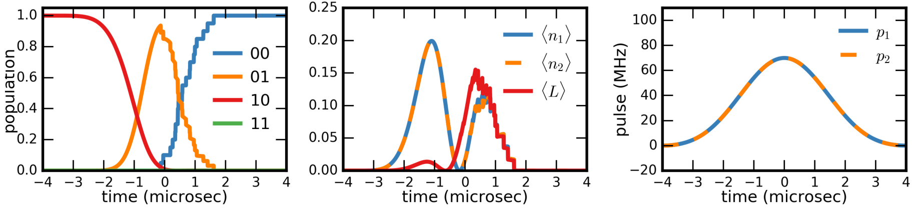

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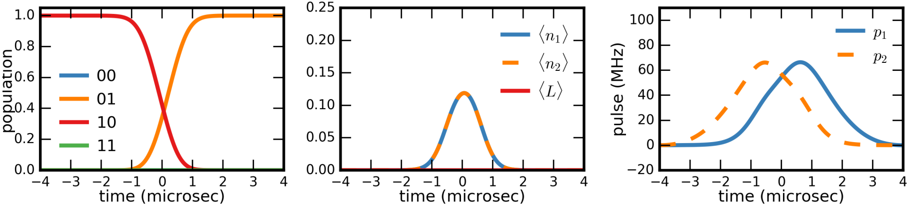


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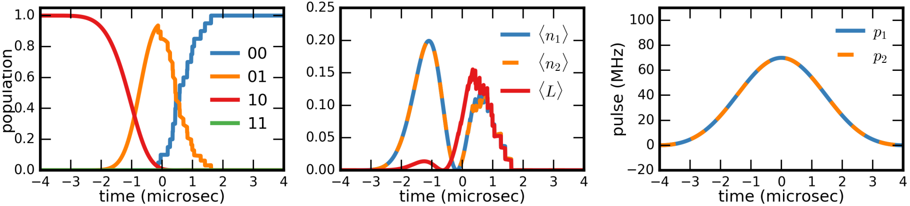
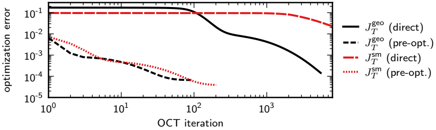


# outlook

- "Hybrid optimization" (combine gradient-free and gradient-based methods); pulse smoothing




[Goerz et al, EPJ Quantum Tech. 2, 21 (2015)]

- Optimize with non-Hermitian Hamiltonian

$$\hat{\mathbf{H}}_{\text{eff}} = \hat{\mathbf{H}} - \frac{i\hbar}{2}\sum_i \hat{\mathbf{L}}_i^\dagger \hat{\mathbf{L}}_i$$

for weak dissipation and unitary target

- Optimize dark state condition $\left\langle \hat{\mathbf{L}}^\dagger \hat{\mathbf{L}} \right\rangle = 0$
[Palao et al, PRA 77, 063412 (2008)]

$\Rightarrow$ Second order Krotov, inhomogeneous bw-propagation
[Reich et al, JCP 136, 104103 (2012)]

# summary & conclusion

- Quantum trajectories are highly scalable approach to simulating open quantum systems (MPI!)
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- Krotov's method allows for trajectory optimization
  (for any large open quantum system, not just networks)
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Thank you!