Optimal Control of Transmon Qubit Gates in the Presence of Decoherence

Quantum Dynamics and Control

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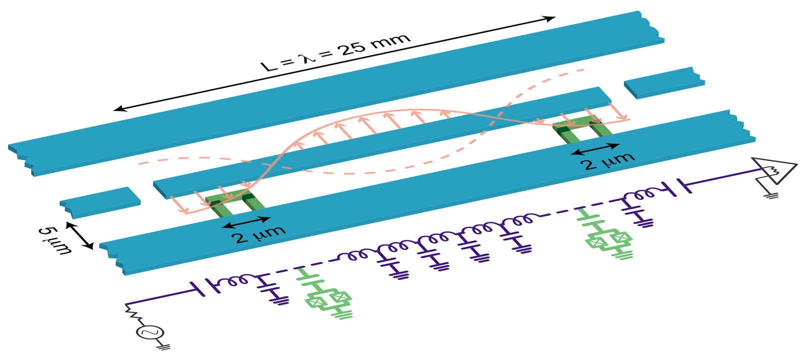
Abstract

We consider two transmon qubits [1] coupled via a cavity bus [2]. The strong coupling of each qubit to the shared cavity modes provides an indirect interaction that in addition to the direct qubit-qubit interaction can be used to implement a two-quantum gate (e.g. CPHASE). Describing the system numerically allows us to take into account an arbitrary number of qubit and cavity excitations. Going beyond the dispersive limit permits the implementation of fast gates, which are necessary to beat decoherence. Optimal control theory (OCT), specifically Krotov's method [3], is used to find microwave pulses that drive the full system in the desired way in the shortest possible amount of time. The complete system Hamiltonian allows for complex dynamics that OCT can fully exploit. We show results from such an optimization for a CPHASE gate, for different pulse durations. We also discuss decoherence and analyze the influence of spontaneous decay of the cavity on the gate fidelity, and give an outlook on how OCT may find robust pathways.

① Two Transmon Qubits Coupled via Cavity Bus



superconducting qubits inside a transmission line resonator, Fig. from [5]

Parameters:

- $\omega_c = 8.3$ GHz
- $\omega_1 = 6.5$ GHz
- $\omega_2 = 6.6$ GHz
- $\alpha_1 = \alpha_2 = 150$ MHz
- $J = 5$ MHz
- $g_1 = g_2 = 100$ MHz
- $|\epsilon(t)| < 50$ MHz (if possible)

$$\hat{\mathbf{H}} = \underbrace{\omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}_{①} + \underbrace{\omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 - (\alpha_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \alpha_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2)}_{②} + \underbrace{J(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_2 + \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_2^\dagger)}_{③} + \underbrace{g_1(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2(\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger)}_{④} + \underbrace{\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger}_{⑤} \tag{1}$$

with ① the cavity harmonic oscillator, ② qubit anharmonic oscillators, ③ direct qubit-qubit coupling, ④ qubit-cavity coupling, and ⑤ cavity coupling to control field $\epsilon(t) \sim E_0 \cos(\omega_L t)$.

Note: Direct qubit-qubit coupling is weak; entanglement is primarily reached indirectly via interaction with the cavity ④.

② Optimization: Krotov Method

We optimize for $\hat{\mathbf{O}}$ =CPHASE by minimizing the functional J containing the gate fidelity F and a running cost ensuring monotonic convergence, with a scaling parameter λ_a and a shape function $S(t)$.

$$J[\{\phi_k\}, \epsilon] = -F[\{\phi_k(T)\}] + \lambda_a \int_0^T \frac{\Delta\epsilon^2(t)}{S(t)} \mathrm{d}t\,, \qquad F = \frac{1}{16}\left|\sum_{k=1}^{4} \left\langle \phi_k \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{U}} \middle| \phi_k \right\rangle\right|^2 \tag{2}$$

with $\Delta\epsilon = \epsilon^{\text{new}} - \epsilon^{\text{old}}$, for $|\phi_k\rangle \in \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, $|\phi_k(t)\rangle = \hat{\mathbf{U}}(t, 0; \epsilon)\,|\phi_k\rangle$.
Pulse update formula [3]:

$$\Delta\epsilon(t) = \frac{S(t)}{\lambda_a} \mathfrak{Im}\left\{ \sum_{k=1}^{4} \left\langle \chi_k^{\text{old}}(t) \middle| \frac{\partial \hat{\mathbf{H}}}{\partial \epsilon^{\text{new}}} \middle| \phi_k^{\text{new}}(t) \right\rangle + \underbrace{\sigma(t) \sum_{k=1}^{4} \left\langle \Delta\phi_k(t) \middle| \frac{\partial \hat{\mathbf{H}}}{\partial \epsilon^{\text{new}}} \middle| \phi_k^{\text{new}}(t) \right\rangle}_{\text{for choices of } F \text{ requiring second order}} \right\}, \tag{3}$$

$$\text{with} \quad |\chi_k(T)\rangle \equiv \frac{\partial F}{\partial \langle \phi_k(T)|} = \frac{1}{16} \sum_{k'=1}^{4} \left\langle \phi_{k'}^{\text{tgt}} \middle| \hat{\mathbf{U}} \middle| \phi_{k'} \right\rangle \left| \phi_k^{\text{tgt}} \right\rangle; \quad \left| \phi_k^{\text{tgt}} \right\rangle \equiv \hat{\mathbf{O}}\,|\phi_k\rangle\,.$$

③ Decoherence

Open system dynamics with a master equation in Lindblad form [4].

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -\mathrm{i}[\hat{\mathbf{H}}, \hat{\rho}] + \kappa \mathcal{D}[\hat{\mathbf{a}}]\hat{\rho} + \sum_{i=1,2} \left[\sum_j \gamma_{i,j} \mathcal{D}[\hat{\mathbf{\Pi}}_{j,j+1}^i]\hat{\rho} + 2\gamma_{\phi,i} \mathcal{D}[\hat{\mathbf{\Pi}}_\epsilon^i]\hat{\rho} + \sum_j \gamma_{i,j}^{th} \mathcal{D}[\hat{\mathbf{\Pi}}_{j+1,j}^i]\hat{\rho} \right] \tag{4}$$

$$\text{with} \quad \mathcal{D}[\hat{\mathbf{A}}]\hat{\rho} = \frac{1}{2}\left(2\hat{\mathbf{A}}\hat{\rho}\hat{\mathbf{A}}^2 - \hat{\mathbf{A}}^+\hat{\mathbf{A}}\hat{\rho} - \hat{\rho}\hat{\mathbf{A}}^+\hat{\mathbf{A}}\right), \qquad \hat{\mathbf{\Pi}}_{n,m}^i = |n\rangle\langle m|\,, \qquad \hat{\mathbf{\Pi}}_\epsilon^i = \sum_n \epsilon_n |n\rangle\langle n|$$

The parameters $\gamma_{i,j}$, $\gamma_{\phi,i}$ and $\gamma_{j,i}^{th}$ are decay, dephasing and leakage rates, respectively. The cavity decay is described by $\kappa = 1/\tau$. For a 3D transmon qubit, lifetime τ can be 20-100 μs.

It is straightforward to write the Krotov update equation (3) for Liouville space, using density matrices instead of states and using Eq. (4) for propagation. It can be shown that it is sufficient to use three density matrices as "basis states":

$$\rho_{ij}^{(1)} = \frac{2(N-i+1)}{N(N+1)}\delta_{ij}\,, \qquad \rho_{ij}^{(2)} = \frac{N^2-2}{N^2}\delta_{i1}\delta_{ji} + \frac{1}{N^2}\delta_{1j} + \frac{1}{N^2}\delta_{i1}\,, \qquad \rho_{ij}^{(3)} = \frac{1}{N}\delta_{ij}\,.$$

Possible choices for a fidelity: $F = \mathrm{tr}\left[\hat{\rho}^{(i)}(T)\hat{\rho}_{\text{tgt}}^{(i)}\right]$, $\quad F = \sum_{ikl} \left| \left(\hat{\rho}^{(i)}(T) - \hat{\rho}_{\text{tgt}}^{(i)} \right)_{kl} \right|^2, \ldots$

Any other distance measure on density matrices may be used.

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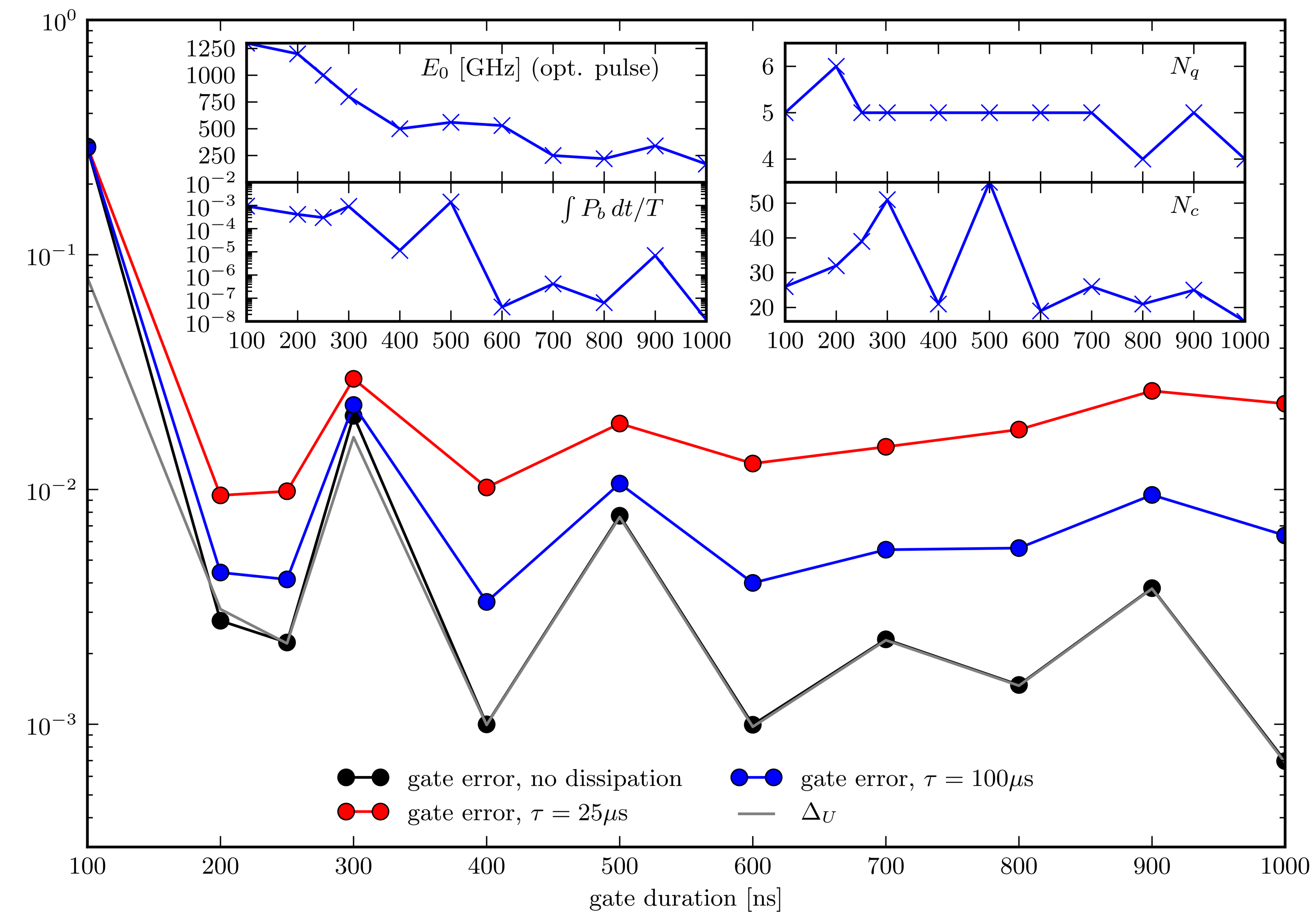
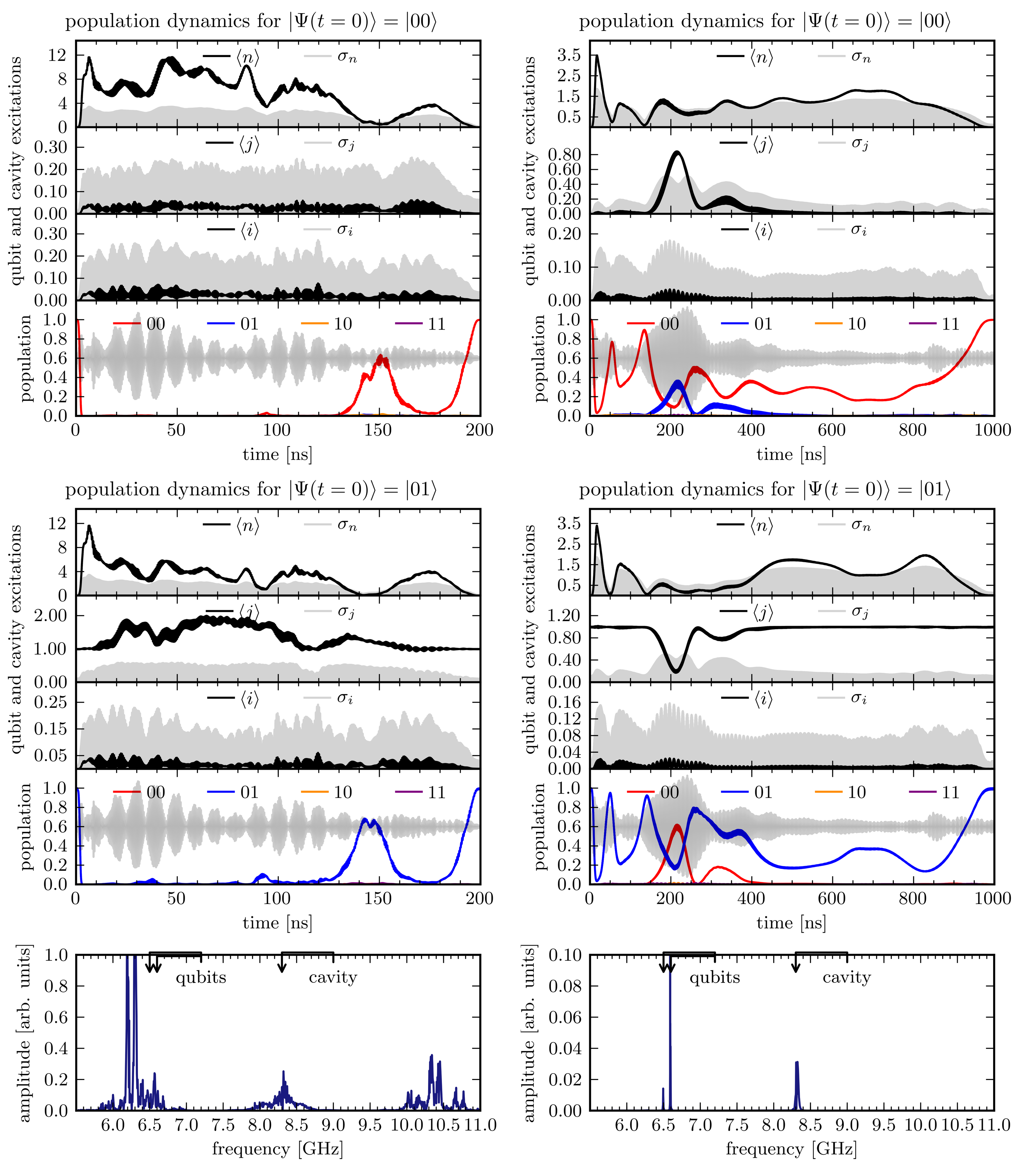
④ Optimization Results (CPHASE)



$\Delta_U = 1 - \mathrm{tr}\left[\hat{\mathbf{U}}\hat{\mathbf{U}}^\dagger\right]$: loss from logical subspace; P_b: Pop. in levels that cannot be resolved from surrounding levels.

- limits on fidelity: pulse duration too short to generate necessary entanglement (100 ns); difficulty to restore cavity ground state if too much excitation
- tradeoff between pulse intensity and durations. $E_0 < 50$ MHz only for very long gate times
- possible explanation for oscillations: population in states that cannot be resolved



Frequencies for transitions from the cavity-dressed logical subspace:

$\lvert 1,1,0\rangle \to \lvert 1,1,1\rangle$: 8.308 GHz;	$\lvert 1,0,0\rangle \to \lvert 1,1,0\rangle$: 6.594 GHz;	$\lvert 0,1,0\rangle \to \lvert 1,0,1\rangle$: 8.210 GHz;
$\lvert 0,1,0\rangle \to \lvert 0,1,1\rangle$: 8.310 GHz;	$\lvert 0,0,0\rangle \to \lvert 0,1,0\rangle$: 6.594 GHz;	$\lvert 1,0,0\rangle \to \lvert 0,1,1\rangle$: 8.409 GHz;
$\lvert 1,0,0\rangle \to \lvert 1,0,1\rangle$: 8.310 GHz;	$\lvert 0,0,0\rangle \to \lvert 1,0,0\rangle$: 6.494 GHz;	$\lvert 1,1,0\rangle \to \lvert 0,2,1\rangle$: 8.110 GHz;
$\lvert 0,0,0\rangle \to \lvert 0,0,1\rangle$: 8.311 GHz;	$\lvert 0,1,0\rangle \to \lvert 1,1,0\rangle$: 6.494 GHz;	$\lvert 1,1,0\rangle \to \lvert 1,0,2\rangle$: 10.026 GHz;
$\lvert 1,1,0\rangle \to \lvert 1,2,0\rangle$: 6.296 GHz;	$\lvert 1,0,0\rangle \to \lvert 0,2,0\rangle$: 6.396 GHz;	$\lvert 0,1,0\rangle \to \lvert 0,0,2\rangle$: 10.029 GHz;
$\lvert 0,1,0\rangle \to \lvert 0,2,0\rangle$: 6.296 GHz;	$\lvert 0,1,0\rangle \to \lvert 2,0,0\rangle$: 6.096 GHz;	$\lvert 1,0,0\rangle \to \lvert 0,0,2\rangle$: 10.128 GHz;
$\lvert 1,0,0\rangle \to \lvert 2,0,0\rangle$: 6.196 GHz;	$\lvert 1,1,0\rangle \to \lvert 0,3,0\rangle$: 5.798 GHz;	$\lvert 1,1,0\rangle \to \lvert 0,1,2\rangle$: 10.125 GHz;
$\lvert 1,1,0\rangle \to \lvert 2,1,0\rangle$: 6.196 GHz;	$\lvert 1,1,0\rangle \to \lvert 3,0,0\rangle$: 5.499 GHz;	$\lvert 1,1,0\rangle \to \lvert 2,0,1\rangle$: 7.910 GHz

⑤ Conclusions & Outlook

- Optimal control successfully finds *fast* gates at fidelities *at the quantum error correction threshold.* Gates are sufficiently fast to (almost) beat decoherence.
- The optimization of a CPHASE gate illustrates the extremely rich dynamics that the Hamiltonian Eq. (1) provides. Other two-qubit quantum gates are possible as well, and may be more robust in the presence of decoherence.
- Pulses may populate a significant number of higher qubit and cavity states, requiring a description beyond an effective two-qubit model. Optimizing from appropriate guess pulses, the number of qubit and cavity levels can be kept reasonably low (e.g. 4-5 qubit levels, 20 cavity levels)
- Optimize in Liouville space, with full decoherence model (Lindblad equation), improving the fidelity under dissipation – if possible to the quantum error correction threshold. This can be done efficiently using the approach presented in section 3. The choice of fidelity may have a significant effect on the optimization success and should be explored systematically.
- Ultimately: Use the local invariants functional [6] in Liouville space to optimize for the two-qubit gate least susceptible to decoherence.