Optimal Control for High-Precision Atom Interferometry

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① Atomic Fountain Interferometer

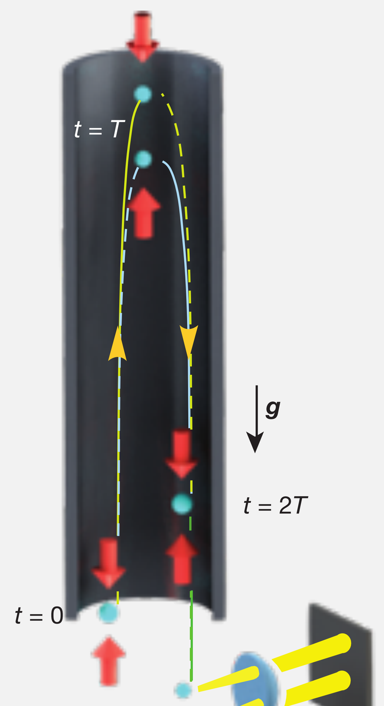
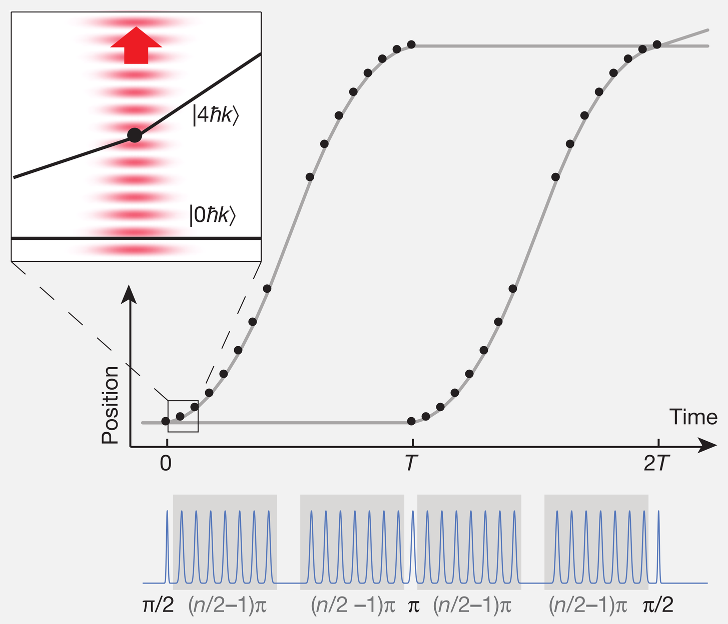


Fig from [1]

10 m atomic fountain at Stanford:
launch BEC cloud of ^{87}Rb atoms.



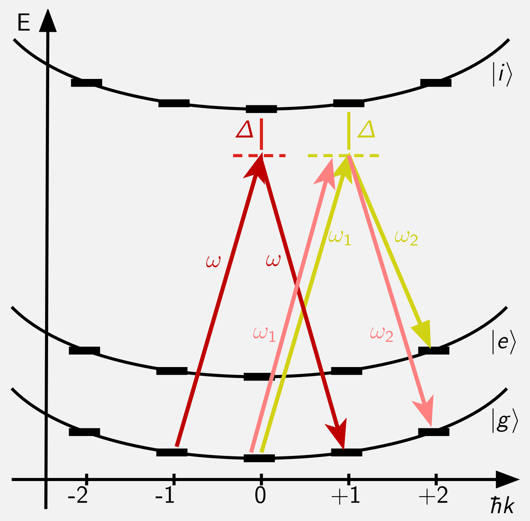
laser pulses implement atomic beamsplitter and mirror

Applications

- Gravitational detection: $\Delta\phi = m a \Delta z T/\hbar$; gravitational wave detector, satellite-based sensing of underground structures
- test of equivalence principle

② Pulse Schemes

Hamiltonian couples momentum and internal state: conservation of angular momentum



- Raman: internal state of atom changes [2]
  adiabatic elimination of $|i\rangle$;
  or rapid adiabatic passage
- Bragg: internal state of atom unchanged
  generalization: different Bragg orders by tuning frequencies [3]
  adiabatic elimination of $|i\rangle$;
  or rapid adiabatic passage
- also: atomic lattice waveguides [4]

Challenges

- require short pulses to avoid Doppler sensitivity
- sequential transitions: accumulating errors
- dephasing from lase intensity dependent Stark shift
- intensity variations across ensemble

⇒ optimal control

References

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[2] Young, Kasevich, and Chu. In Berman, "Atom Interferometry" (Academic Press, 1997)
[3] Berman and Bian, *Phys. Rev. A* **55**, 4382 (1997)
[4] Kovachy et al, *Phys. Rev. A* **82**, 013638 (2010)
[5] Doria, Calarco, and Montangero, *Phys. Rev. Lett.* **106**, 190501 (2011)
[6] Reich et al. *J. Chem. Phys.* **136**, 104103 (2012)
[7] Khaneja et al. *J. Magnet. Res.* **172**, 296 (2005)
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[9] Malinovky, Berman, *Phys. Rev. A* **68**, 023610 (2003)

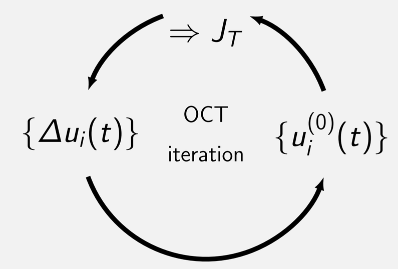
③ Optimal Control Methods

goal: find controls that minimize functional for reaching the target state

$$J_T = 1 - \mathfrak{Re}\left\langle \Psi(T) \,\middle|\, \Psi^{\mathrm{tgt}} \right\rangle; \quad |\Psi(T)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^T \mathrm{d}t \hat{\mathrm{H}}(\{u_i(t)\})\right]|\Psi(0)\rangle$$

iterative procedure:

- start from guess pulse
- simulate dynamics; calculate pulse update $\Delta u(t)$
- updated ("optimized") pulse is guess for next iteration



Gradient-free: Nelder-Mead Simplex

- systematically vary control parameters to "roll down the landscape"
- numerically cheap:
  only need to evaluate J_T
- Only works for a small number of free control parameters!

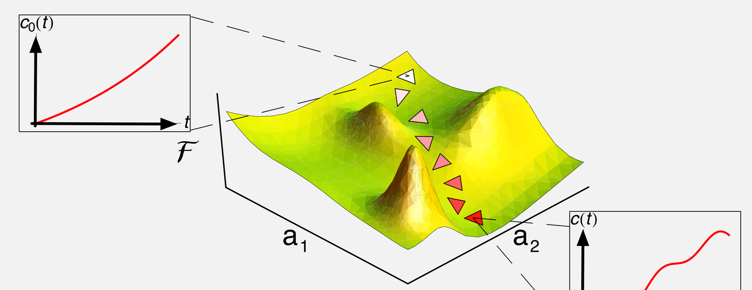


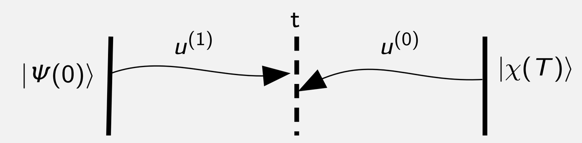
Fig adapted from [5]

Gradient-based: Krotov's method [6]

$$\text{auxiliary functional } J = J_T + \sum_i \frac{\lambda_i}{S_i(t)} \int_0^T \overbrace{\left|u_i^{(1)}(t) - \underbrace{u_i^{(0)}(t)}_{u_i^{\mathrm{ref}}}\right|^2}^{|\Delta u_i(t)|^2} \mathrm{d}t$$

update $\Delta u_i(t) = \frac{S_i(t)}{\lambda_i}\mathfrak{Im}\left\langle \chi^{(0)}(t) \,\middle|\, \frac{\partial H}{\partial u_i(t)} \,\middle|\, \Psi^{(1)}(t) \right\rangle$;

boundary cond. $\left|\chi^{(0)}(T)\right\rangle = \frac{\partial J_T}{\partial\langle\Psi^{\mathrm{tgt}}|}$

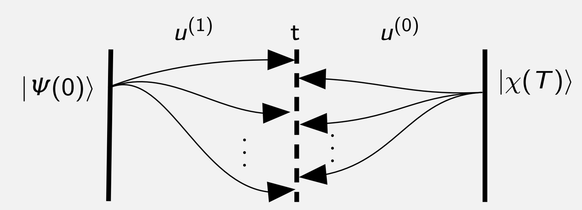


alternative method: gradient ascent (GRAPE) [7]: $\Delta u_i(t) \propto \frac{\partial J_T}{\partial u_i(t)}$

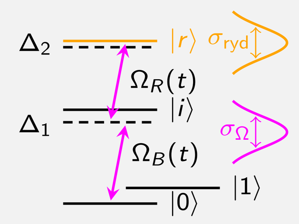
hybrid approach for best results:
pre-optimize with Nelder-Mead first, then continue with gradient search.

④ Robustness through Ensemble Optimization

idea: sample noise-realizations of Hamiltonian

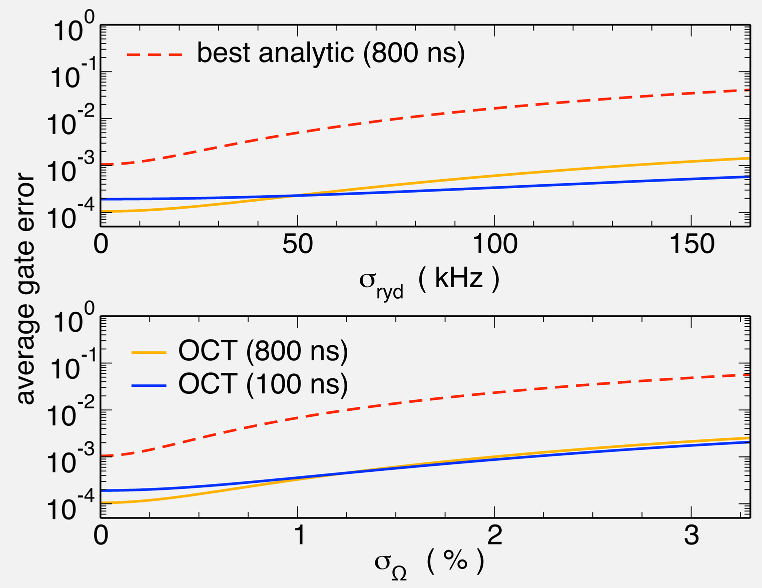


example: Rydberg gate [8]



dominant noise sources:

- fluctuation of Rydberg level (stray magnetic fields)
- fluctuation in pulse amplitude ⇒ dipole value



compared to best analytical scheme:
optimal control reduces gate duration from 800 ns to 100 ns, and is *order of magnitude more robust.*

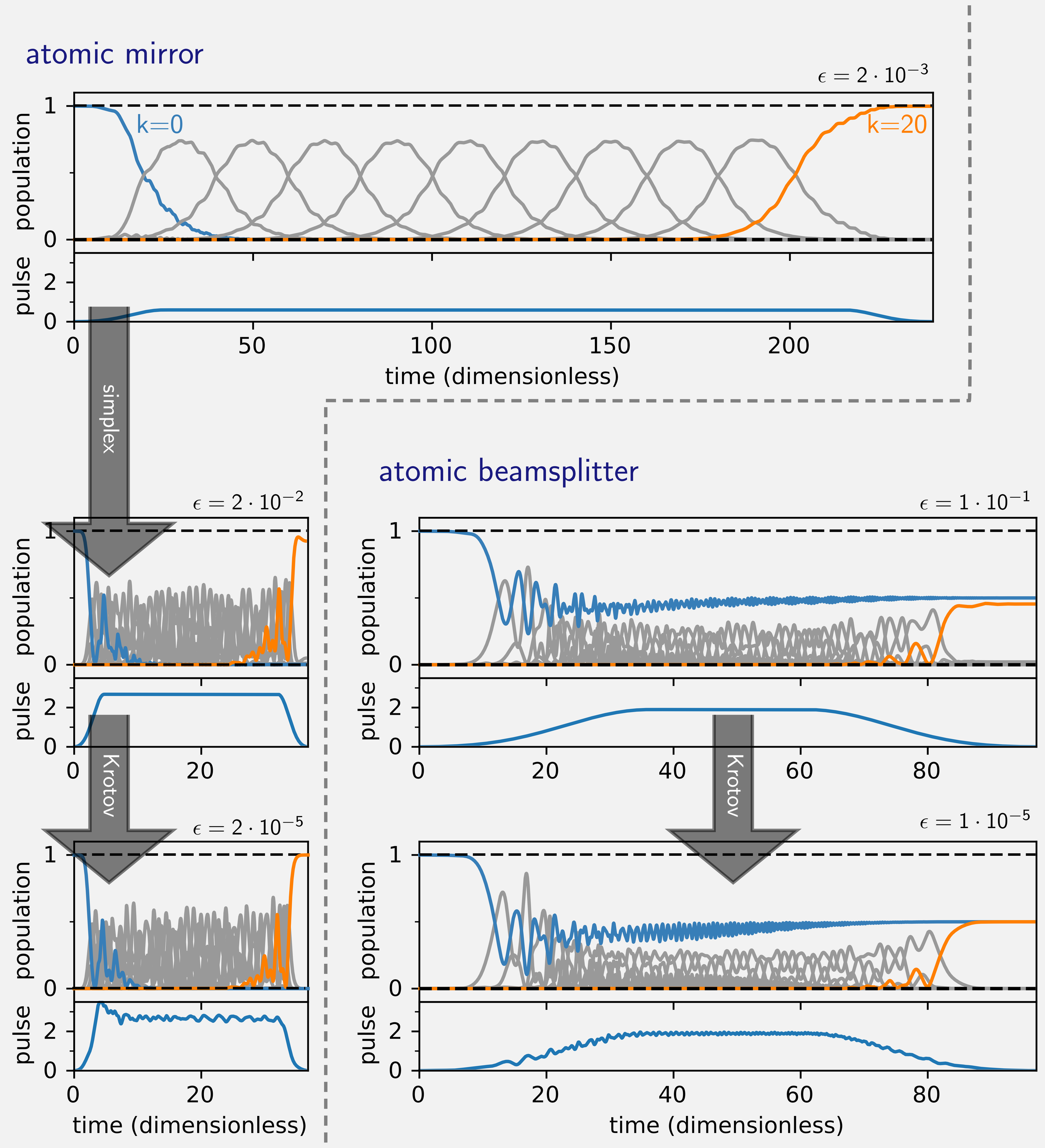
⑤ Optimization Results for Simplified Model

using model from Ref [9]

$$\hat{\mathrm{H}}(t) = \left\{\begin{matrix} \ddots & \ddots & 0 & & \\ \ddots & E_{-1}(t)/\hbar & -\Omega_{-n}(t) & 0 & \\ 0 & -\Omega^*_{-n}(t) & E_0(t)/\hbar & -\Omega_{+n}(t) & 0 \\ & 0 & -\Omega^*_{+n}(t) & E_{+1}(t)/\hbar & \ddots \\ & & 0 & \ddots & \ddots \end{matrix}\right\};$$

$$\Omega_n(t) = \Omega(t) \quad \text{(envelope)}$$
$$E_n(t) = \hbar\left(n^2\omega_k + n\alpha t\right)$$
with chirp rate α

1. pre-optimization (simplex): assuming Blackman shape; vary switch-on, pulse duration, amplitude, chirp rate
2. Krotov: optimize pulse envelope



⑥ Outlook

- Use full model with additional levels; no adiabatic elimination
- Ensemble optimization to address *challenges* (see ②)

Modeling: QNET computer algebra system `https://qnet.readthedocs.io`

Simulation and optimization: `https://qdyn-library.net`