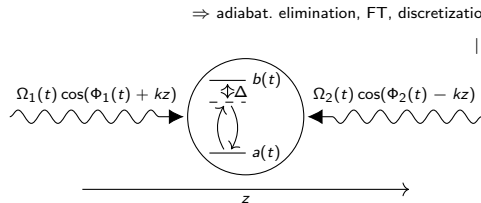
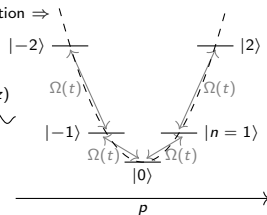


Analytic Pulse Schemes

(a) coordinate space



(b) momentum space



effective field:

$$\Omega(t) = \frac{\Omega_1(t)\Omega_2(t)}{4\Delta}$$

$$\phi(t) = \Phi_1(t) - \Phi_2(t)$$

$$\omega_k = \frac{2\hbar k^2}{m} \quad (\text{recoil frequency})$$

$$\beta = \frac{mv_0}{2\hbar k} \quad (\text{initial velocity})$$

Hamiltonian in momentum space:

$$\hat{H} = \sum_n \left[(E_n^{(0)}(t) + 2n\beta\hbar\omega_k) |n\rangle\langle n| - \mu S(t) (|n\rangle\langle n+1| + \text{H.c.}) \right] \quad \text{with} \quad E_n^{(0)}(t) = n^2\hbar\omega_k + n\hbar\dot{\phi}(t)$$

Robustness landscape: μ (pulse amplitude), β (initial velocity)

① Train of Rabi pulses:

$$\dot{\phi}(t) = \omega_L = -\omega_k(2n_0 + 1)$$

drives resonant Rabi-cycling of transition $n_0 \leftrightarrow n_0 + 1$

② Rapid Adiabatic Passage (RAP) [1]:

$$\dot{\phi}(t) = -\alpha\omega_k(t - t_c)$$

sweeps over resonances at intervals $\tau_B = 2/|\alpha|$. Choose offset t_c to select transition and compensate switch-on/-off

Control Conditions

① Splitting:

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} (|0\rangle + e^{i\Delta\phi} |N_S\rangle)$$

② ⑤ Amplification:

$$|0\rangle \rightarrow |0\rangle, |N_S\rangle \rightarrow e^{i\phi_+} |N_{\max}\rangle$$

After ②: kick

$$|N_{\max}\rangle \rightarrow e^{i\phi_{\text{kick}}} |N_{\max}\rangle$$

③ ⑥ De-amplification:

$$|0\rangle \rightarrow |0\rangle, |N_{\max}\rangle \rightarrow e^{i\phi_-} |N_S\rangle$$

④ SWAP:

$$|0\rangle \leftrightarrow |N_S\rangle$$

⑦ Recombination:

$$|0\rangle \rightarrow \frac{e^{i\phi_{00}}}{\sqrt{2}} (|0\rangle + e^{i\Delta\phi} |1\rangle), |N_S\rangle \rightarrow \frac{e^{i\phi_{10}}}{\sqrt{2}} (|0\rangle + e^{i(\Delta\phi+\pi)} |1\rangle)$$

Results in

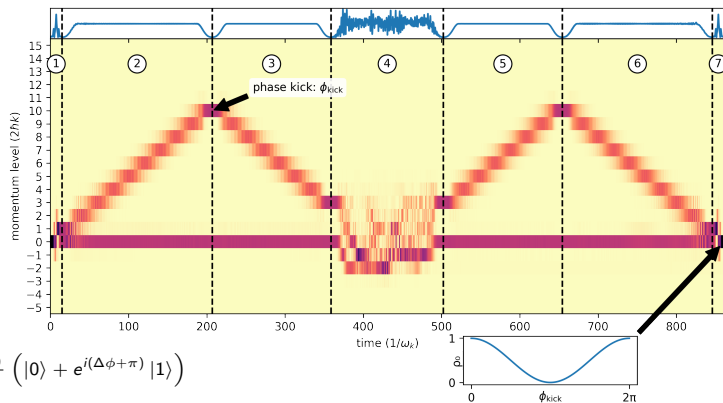
$$\rho_0 = \frac{e^{i\Delta\phi} e^{-i\Delta\phi} e^{i\phi_{\text{kick}}}}{4} + \frac{1}{2} + \frac{e^{-i\Delta\phi} e^{i\Delta\phi} e^{-i\phi_{\text{kick}}}}{4}$$

Required kick response:

$$\rho_0 = \cos^2\left(\frac{\phi_{\text{kick}}}{2}\right) = \frac{1}{2} + \frac{\cos(\phi_{\text{kick}})}{2}, \quad p_1 = 1 - \rho_0$$

■ Phases ϕ_+ , ϕ_- cancel out! → Final recombination only has to compensate $\Delta\phi$ of initial splitter!

■ For $N_S = 1$, same pulse can split and recombine! – $\Delta\phi$ drops out!



Robustness

① Fringe Contrast

$$I_{FC} = \frac{\max \rho_0 - \min \rho_0}{\max \rho_0 + \min \rho_0}$$

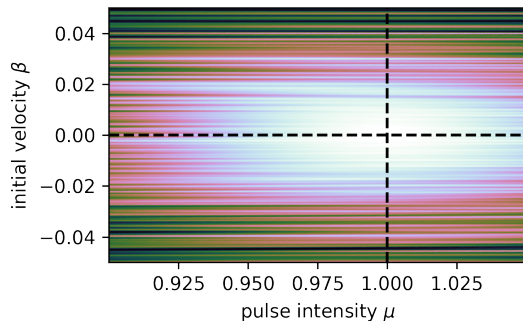
② Zero-Phase Population

$$I_0 = \rho_0(\phi_{\text{kick}=0})$$

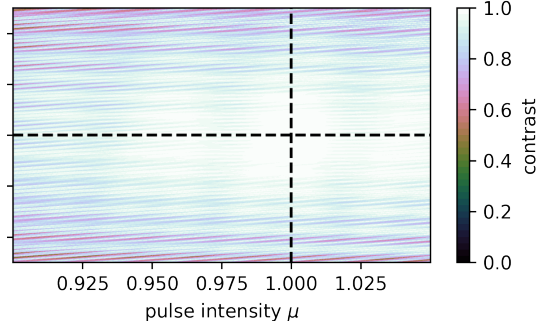
⇒ Total Contrast

$$I_{\text{total}} = I_{FC} I_0$$

Rabi scheme



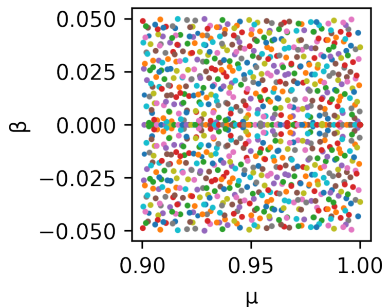
RAP scheme



Ensemble Control for Robustness

Optimal Control Theory

- Iteratively minimize optimization functional J
- Use Krotov's method [2] to find $S(t)$ that monotonically decreases J in each iteration
- Optimize each of the 7 stages individually



Ensemble optimization

- Optimize simultaneously for average of ensemble (μ, β) [3]
- *rotating* ensemble: switch ensemble every 1000 iterations

Population transfer functional: [4]

$$J_{\text{pop}} = \frac{1}{2M} \sum_{(\mu, \beta)} \sum_k \sum_n \left(\left| \langle \phi_k^{(\mu, \beta)}(T) | n \rangle \right|^2 - \left| \langle \phi_k^{\text{tgt}} | n \rangle \right|^2 \right)^2$$

Square-modulus gate functional:

$$J_{\text{sm}} = 1 - \left| \frac{1}{MK} \sum_{(\mu, \beta)} \sum_{k=1}^K \langle \phi_k^{\text{tgt}} | \phi_k^{(\mu, \beta)}(T) \rangle \right|^2.$$

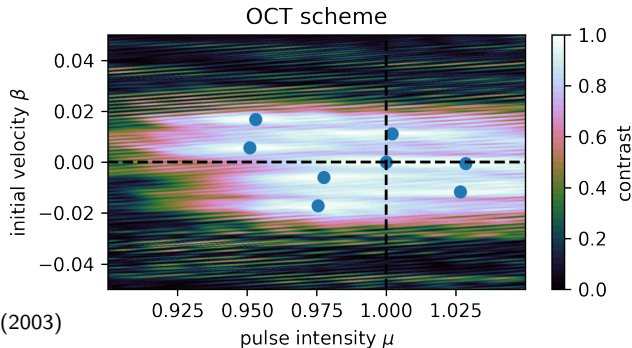
Conclusions & Outlook

Summary

- Shown robustness of RAP scheme over traditional Rabi scheme
- Derived general control conditions for an atomic fountain interferometer
- Formulated population transfer functional
- Ensemble optimization for robustness

References

- [1] Malinovsky, Berman. Phys. Rev. A 68, 023610 (2003)
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- [2] Goerz *et al.*, SciPost Phys. 7, 80 (2019)
<https://doi.org/10.21468/scipostphys.7.6.080>
- [3] Goerz *et al.*, Phys. Rev. A 90, 032329 (2014)
<https://doi.org/10.1103/PhysRevA.90.032329>
- [4] Goerz *et al.*, in Proc. SPIE 11700 (2021)
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Outlook

- Extend ensemble optimization over entire landscape
- Frequency and amplitude constraints