Optimal Control of a Sagnac Tractor Atom Interferometer

Michael H. Goerz[1],
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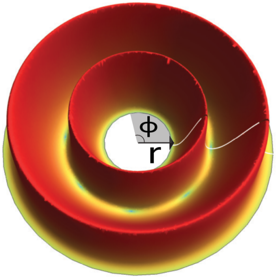
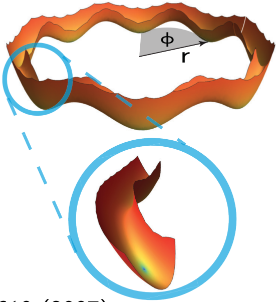
DAMOP Meeting 2023

Pinwheel Optical Lattice





Franke-Arnold *et al.* Opt. Express 15, 8619 (2007)

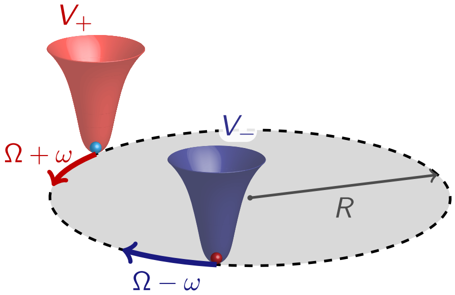
Rubidium-87 hyperfine levels

$$|+\rangle \equiv |F = 1, m_F = 0\rangle$$
$$|-\rangle \equiv |F = 2, m_F = 0\rangle$$

Make it spin-dependent!

Raithel *et al.* Quantum Sci. Technol. 8 (2023)

Rotating Tractor Interferometer



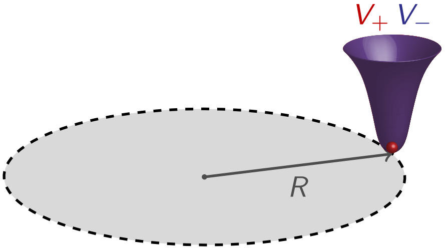
B. Dash *et al.* "Rotation sensing using tractor atom interferometry" (in preparation)

$$\hat{\mathrm{H}}_\pm = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial\theta^2} + \underbrace{V_0 \cos\left[m(\theta + \phi_\pm(t))\right]}_{=V_\pm(\theta,t)}$$

$$\phi_\pm(t) = \int_0^t \omega_\pm(t')\,dt' = \int_0^t \left(\Omega \pm \omega(t')\right)\,dt'$$

Adiabatic Dynamics

$t = 0$ ms



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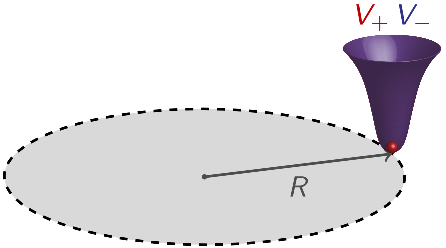
$$\phi_\pm(t) = \int_0^t \omega_\pm(t')\,dt' = \int_0^t \left(\Omega \pm \omega(t')\right)\,dt'$$

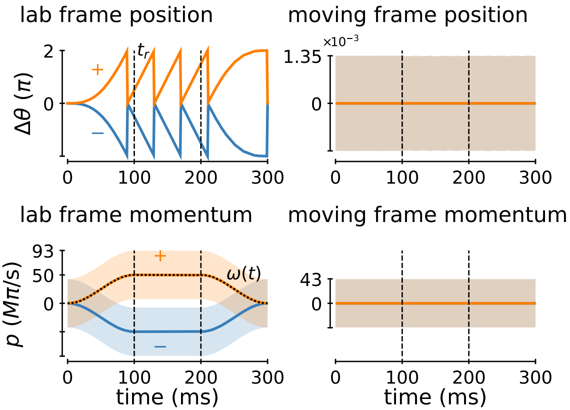
$$\omega(t) = \begin{cases} \omega_0 \sin^2\left(\frac{\pi t}{2t_r}\right) & 0 \leq t < t_r \\ \omega_0 & t_r \leq t < t_r + t_{\text{loop}} \\ \omega_0 \cos^2\left(\frac{\pi t'}{2t_r}\right) & T - t_r \leq t \leq T \end{cases}$$

Adiabatic Dynamics

$t = 300$ ms



$$\omega(t) = \begin{cases} \omega_0 \sin^2\left(\frac{\pi t}{2 t_r}\right) & 0 \le t < t_r \\ \omega_0 & t_r \le t < t_r + t_{\text{loop}} \\ \omega_0 \cos^2\left(\frac{\pi t'}{2 t_r}\right) & T - t_r \le t \le T \end{cases}$$



Interferometric Response

$$\Delta\Phi_S = \frac{4m\Omega A}{\hbar}, \quad A = \frac{R^2}{2} \underbrace{\int_0^T \omega(t')dt'}_{=n\pi}$$

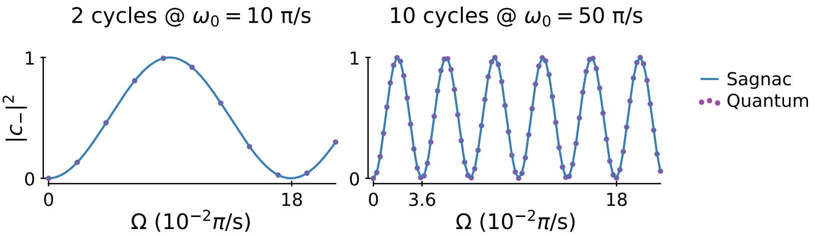
$$|c_\pm|^2 = \frac{1}{2} \pm \frac{1}{2}\mathrm{Re}\left[\eta e^{-i\Delta\Phi}\right] \quad \rightarrow \quad |c_-|^2 = \frac{1}{2} - \frac{\cos\Delta\Phi}{2} = \sin^2\left(\frac{\Delta\Phi}{2}\right)$$

$\eta = \langle\Psi_-(\theta, T)|\Psi_+(\theta, T)\rangle = 1$ if adiabatic

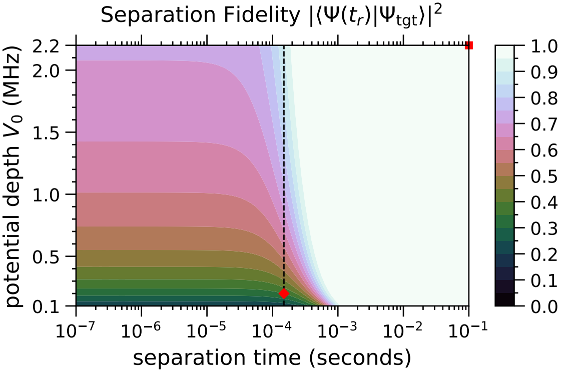
Interferometric Response

$$\Delta\Phi_S = \frac{4m\Omega A}{\hbar}, \quad A = \frac{R^2}{2} \underbrace{\int_0^T \omega(t')dt'}_{=10\pi}$$

$$|c_\pm|^2 = \frac{1}{2} \pm \frac{1}{2}\mathrm{Re}\left[\eta e^{-i\Delta\Phi}\right] \quad \rightarrow \quad |c_-|^2 = \frac{1}{2} - \frac{\cos\Delta\Phi}{2} = \sin^2\left(\frac{\Delta\Phi}{2}\right)$$



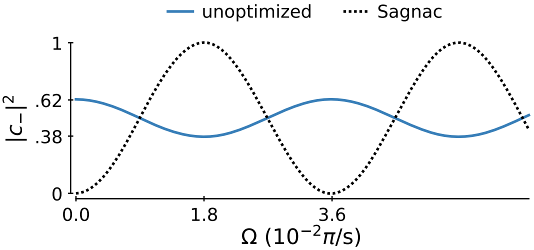
Non-Adiabatic Dynamics



$$\omega(t) = \begin{cases} \omega_0 \sin^2\left(\frac{\pi t}{2 t_r}\right) & 0 \leq t < t_r \\ \omega_0 & t_r \leq t < t_r + t_{\text{loop}} \\ \omega_0 \cos^2\left(\frac{\pi t'}{2 t_r}\right) & T - t_r \leq t \leq T \end{cases}$$

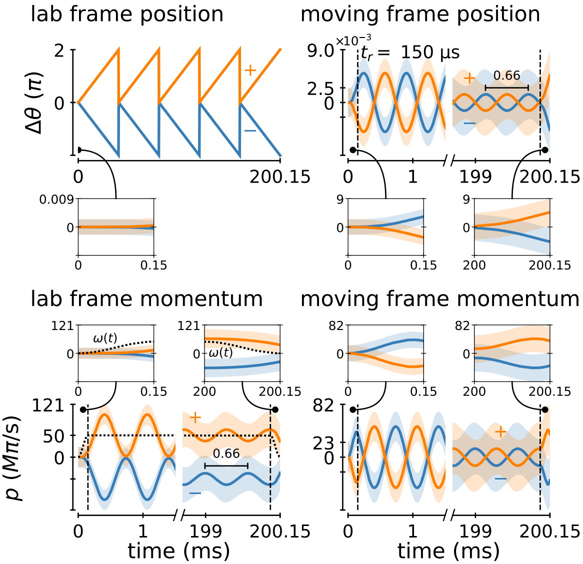
$|\Psi_{\text{tgt}}\rangle$ = ground state of moving potential

Non-Adiabatic Dynamics



$$\Delta\Phi_S = \frac{4m\Omega A}{\hbar}, \quad A = \frac{R^2}{2}\cdot 10\pi$$

$$|c_-|^2 = \frac{1}{2} - \frac{1}{2}\mathrm{Re}\left[\eta e^{-i\Delta\Phi}\right]$$



Control Problem

$$\omega(t) = \begin{cases} \omega_{\text{opt}}(t) & 0 \leq t < t_r \\ \omega_0 & t_r \leq t < t_r + t_{\text{loop}} \\ \omega_{\text{opt}}(t') & T - t_r \leq t \leq T \end{cases}$$

Find $\omega_{\text{opt}}(t)$ for short t_r so that

$$\Psi(\theta, t = 0) \rightarrow \Psi_{\text{tgt}}(\theta, t = t_r)$$

where $|\Psi_{\text{tgt}}\rangle$ = ground state of moving potential

Optimization with QuantumControl.jl

JuliaQuantumControl

Julia Framework for Quantum Optimal Control

19 followers https://juliaquantumcontrol.github.i...

Overview | Repositories 15 | Discussions | Projects | Packages | People 3

README.md

A Julia Framework for Quantum Optimal Control.

docs stable | docs dev

The JuliaQuantumControl organization collects packages implementing a comprehensive collection of methods of open-loop quantum optimal control.

Quantum optimal control theory attempts to steer a quantum system in some desired way by finding optimal control parameters or control fields inside the system Hamiltonian or Liouvillian. Typical control tasks are the preparation of a specific quantum state or the realization of a logical gate in a quantum computer. Thus, quantum control theory is a critical part of realizing quantum technologies, at the lowest level. Numerical methods of *open-loop* quantum control (methods that do not involve measurement feedback from a physical quantum device) such as Krotov's method and GRAPE address the control problem by simulating the dynamics of the system and then iteratively improving the value of a functional that encodes the desired outcome.

github.com/JuliaQuantumControl

umControl that implement specific individual methods are combined in the high-level package `QuantumControl.jl` For normal usage, i.e., outside of development within the JuliaQuantumControl organization, it should

People

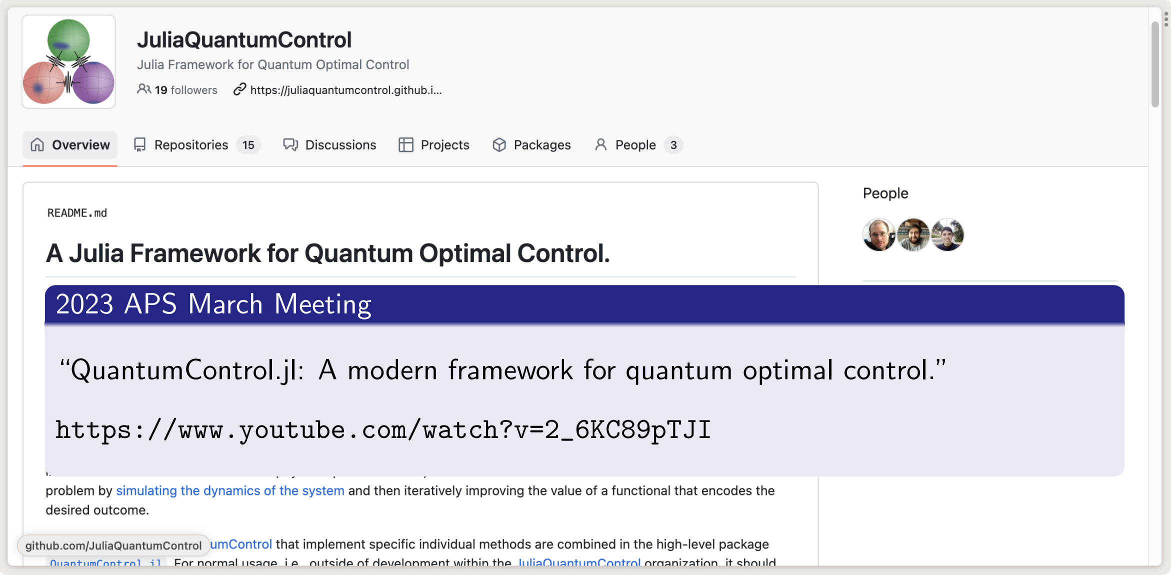
Top languages

Julia Makefile

Most used topics

julia quantum grape optimal-control quantum-computing

Optimization with QuantumControl.jl



2023 APS March Meeting

"QuantumControl.jl: A modern framework for quantum optimal control."

https://www.youtube.com/watch?v=2_6KC89pTJI

Optimization with QuantumControl.jl

2023-05-17_OCT_tr=150μs_V0=0.2MHz_R=26μm_ω=50πps.ipynb

File Edit View Run Kernel Tabs Settings Help

Code git Interface Julia 1.8.5

[38]:
```
objective = Objective(
    initial_state=INITIAL_STATE,
    target_state=TARGET_STATE,
    generator=set_guided_control(HAMILTONIAN, TIME_GRID)
);
δω = get_controls(objective.generator)[1];
```
Last executed at 2023-06-04 15:27:22 in 711ms

[39]:
```
problem = ControlProblem(;
    objectives=[objective], tlist=TIME_GRID,
    J_T=J_T_sm, prop_method=:splitprop, verbose=false,
    pulse_options=IdDict(δω => Dict(:lambda_a => 1e6, :update_shape => t -> 1.0)),
    check_convergence=res -> begin ((res.J_T < 1e-8) && (res.converged = true) && (res.message = "J_T < 10⁻⁸")) e
);
```
Last executed at 2023-06-04 15:27:22 in 284ms

[40]:
```
res = @optimize_or_load("./data/$NAME.jld2", problem; method=:krotov, iter_stop=400, force=true)
```
Last executed at 2023-06-04 15:28:04 in 42.53s

```
 iter.        J_T   ∫gₐ(t)dt          J        ΔJ_T         ΔJ    secs
     0   6.48e-01   0.00e+00   6.48e-01        n/a         n/a     0.9
     1   6.38e-01   4.99e-03   6.43e-01  -1.01e-02   -5.09e-03     5.3
     2   6.27e-01   5.13e-03   6.33e-01  -1.04e-02   -5.23e-03     0.1
     3   6.17e-01   5.28e-03   6.22e-01  -1.07e-02   -5.38e-03     0.1
     4   6.06e-01   5.42e-03   6.11e-01  -1.09e-02   -5.53e-03     0.1
```

Optimization with QuantumControl.jl

2023-05-17_OCT_tr=150µs_V0=0.2MHz_R=26µm_ω=50πps.ipynb

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);
δω = get_controls(objective.generator)[1];
```
Last executed at 2023-06-04 15:27:22 in 711ms

[39]:

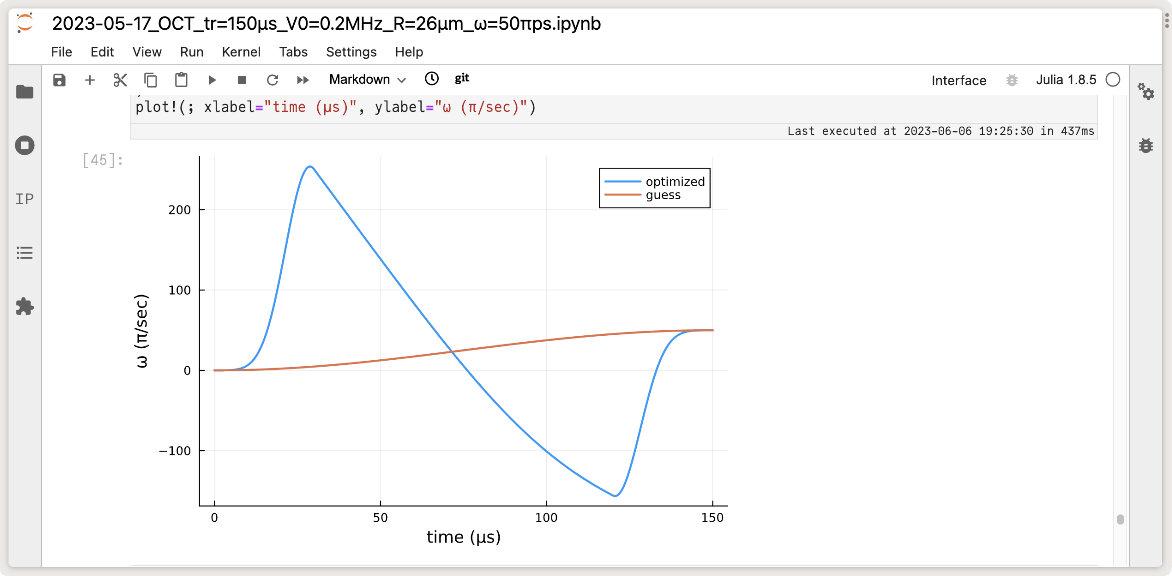
Guided Control

$$\omega_{\text{opt}}(t) = \omega(t) + S(t)\delta\omega(t)$$

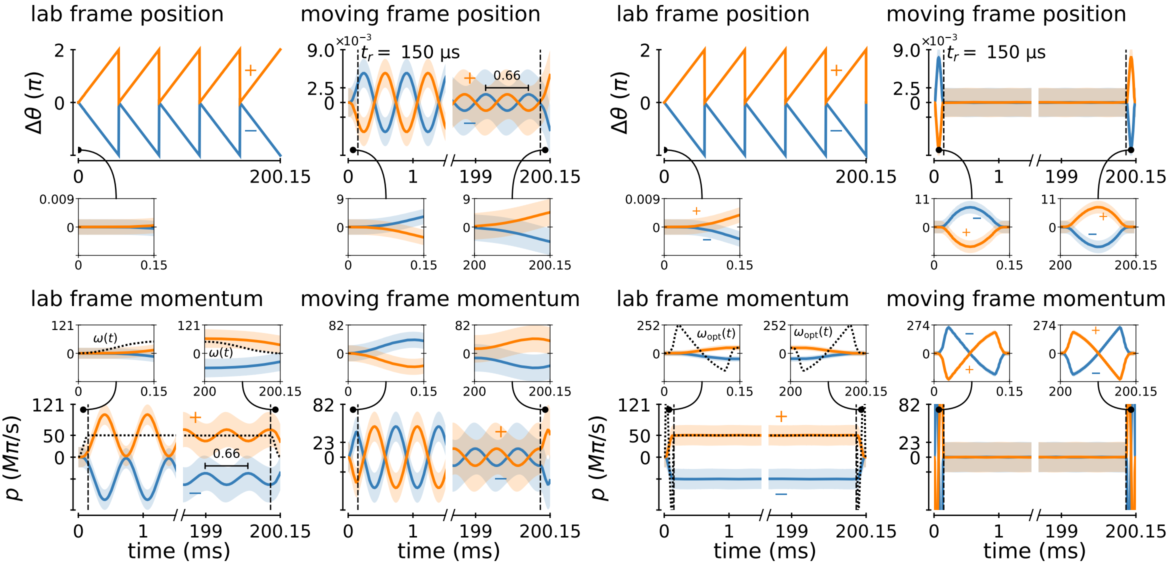
[40]:
```
res = @optimize_or_load("./data/$NAME.jld2", problem; method=:krotov, iter_stop=400, force=true)
```
Last executed at 2023-06-04 15:28:04 in 42.53s

iter.	J_T	∫gₐ(t)dt	J	ΔJ_T	ΔJ	secs
0	6.48e-01	0.00e+00	6.48e-01	n/a	n/a	0.9
1	6.38e-01	4.99e-03	6.43e-01	-1.01e-02	-5.09e-03	5.3
2	6.27e-01	5.13e-03	6.33e-01	-1.04e-02	-5.23e-03	0.1
3	6.17e-01	5.28e-03	6.22e-01	-1.07e-02	-5.38e-03	0.1
4	6.06e-01	5.42e-03	6.11e-01	-1.09e-02	-5.53e-03	0.1

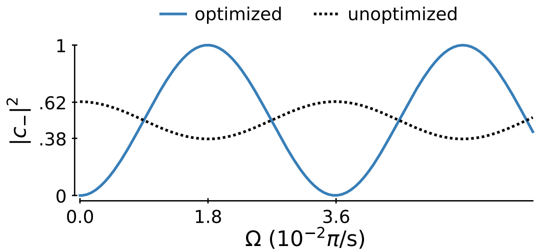
Optimization with QuantumControl.jl



Optimized Dynamics



Optimized Dynamics



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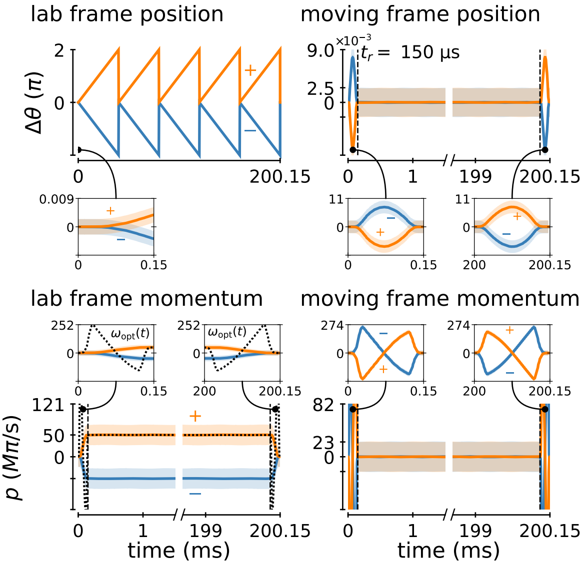

Conclusions

Tractor Atom Interferometer

- Pinwheel optical lattice with freely tuneable angular velocity
- Can be made spin-dependent (Rubidium hyperfine levels)
- Continuous confinement guarantees closure of interferometer (if adiabatic)
- Highly scalable due to multi-pass design

Optimal Control

- Control problem: non-adiabatically go to moving-lattice ground state
- Optimization with QuantumControl.jl
  `https://github.com/JuliaQuantumControl`
- "Throw and catch" solution restores full contrast

Thank You