Efficient Optimal Control for a Unitary Operation under Dissipative Evolution

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Introduction: numerical optimal control

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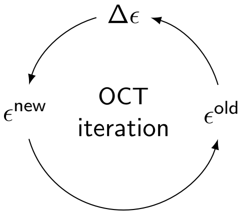

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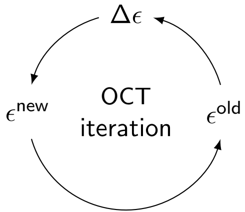

- "optimal": not limited to simple intuitive schemes, operate at the quantum speed limit

Gate optimization

$$\text{CPHASE} = \text{diag}(-1, 1, 1, 1)$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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Goal: Maximize

$$F = \frac{1}{d} \sum_{i=1}^{d} \mathfrak{Re} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{U}}(T, 0, \epsilon) \middle| \Psi_i \right\rangle$$

Two-qubit gates: $d = 4$

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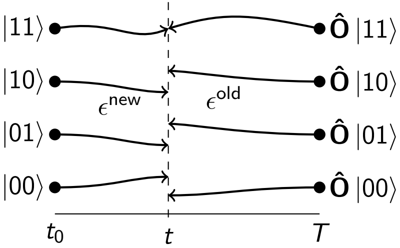
$$\Delta\epsilon(t) \propto \left\langle \chi(t) \middle| \partial_\epsilon \hat{\mathbf{H}} \middle| \Psi(t) \right\rangle$$



OCT for open quantum systems

In the real world: decoherence

OCT for open quantum systems

$$\hat{\rho}(T) = \mathcal{D}(\hat{\rho}(0)); \qquad \text{for example } \frac{\partial \hat{\rho}}{\partial t} = \frac{i}{\hbar}[\hat{\mathbf{H}}, \hat{\rho}] + \mathcal{L}_D(\hat{\rho})$$

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Lift $F = \frac{1}{d} \sum_{i=1}^{d} \mathfrak{Re} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{P}} \hat{\mathbf{U}}(T, 0, \epsilon) \hat{\mathbf{P}} \middle| \Psi_i \right\rangle$ to Liouville space.

Kallush & Kosloff, Phys. Rev. A 73, 032324 (2006),
Ohtsuki, New J. Phys. 12, 045002 (2010)
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$$\Rightarrow F = \frac{1}{d^2} \sum_{j=1}^{d^2} \operatorname{tr}\left[\hat{\mathbf{O}} \hat{\rho}_j(0) \hat{\mathbf{O}}^\dagger \, \hat{\rho}_j(T)\right]$$

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$$\hat{\rho}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\rho}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\rho}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \dots$$

d^2 matrices to propagate! (16 for two-qubit gate)

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Claim

We only need to propagate **three** matrices (independent of d), instead of d^2.

A reduced set of density matrices

No need to characterize the full dynamical map! – much less information required to assess how well a desired unitary is implemented

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$$\hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A reduced set of density matrices

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1. **Do we stay in the logical subspace?**
2. **Are we unitary, and if yes, did we implement the right gate?**

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E.g. $\hat{\mathbf{O}} = \text{diag}(-1, 1, 1, 1)$;
For $\hat{\mathbf{U}} = \text{diag}(e^{i\phi_{00}}, e^{i\phi_{01}}, e^{i\phi_{10}}, e^{i\phi_{11}})$
using just $\hat{\rho}_1$ will not distinguish $\hat{\mathbf{U}}$ from $\hat{\mathbf{O}}$. ($\hat{\mathbf{U}}\hat{\rho}_1\hat{\mathbf{U}}^\dagger = \hat{\mathbf{O}}\hat{\rho}_1\hat{\mathbf{O}}^\dagger = \hat{\rho}_1$)

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Efficient gate optimization in Liouville space

Optimization States

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populations　　　　phases　　　　subspace

Functional

$$J_T = 1 - \sum_{j=1}^{3} \frac{w_j}{\mathrm{tr}[\hat{\rho}_j^2(0)]} \,\mathrm{tr}\left[\hat{\mathbf{O}}\hat{\rho}_j\hat{\mathbf{O}}^\dagger \mathcal{D}[\hat{\rho}_j]\right]$$

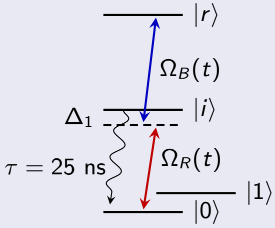
- Allow for different weights ($\sum w_j = 1$)
- $J_T = 0$ iff for all $\hat{\rho}_j$: $\mathcal{D}[\hat{\rho}_j] \equiv$ target state
  $\Rightarrow$ implemented unitary gate $\hat{\mathbf{O}}$.

Example 1

Optimization of a Diagonal Gate using Rydberg Atoms

Two trapped neutral atoms

Single-qubit Hamiltonian



In the RWA:

$$\hat{\mathbf{H}}_{1q} = \begin{pmatrix} 0 & 0 & \frac{1}{2}\Omega_R(t) & 0 \\ 0 & E1 & 0 & 0 \\ \frac{1}{2}\Omega_R(t) & 0 & \Delta_1 & \frac{1}{2}\Omega_B(t) \\ 0 & 0 & \frac{1}{2}\Omega_B(t) & 0 \end{pmatrix}$$

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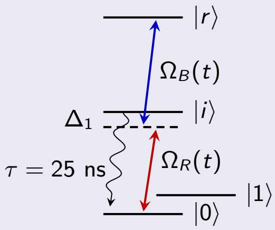

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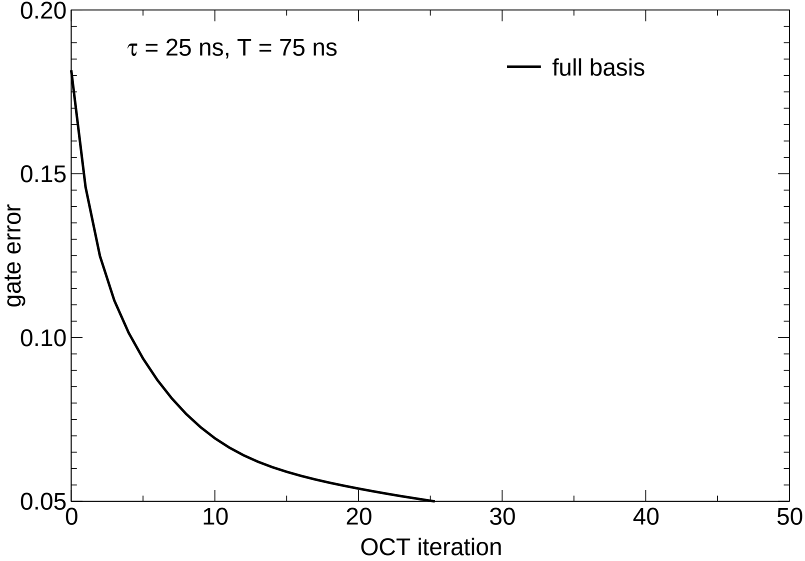
Two-qubit Hamiltonian

$$\hat{\mathbf{H}}_{2q} = \hat{\mathbf{H}}_{1q} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{H}}_{1q} - U \left|rr\right\rangle \left\langle rr\right|$$

Dipole-dipole interaction when both atoms in Rydberg state.

Only diagonal gates!

Optimization of a Rydberg gate



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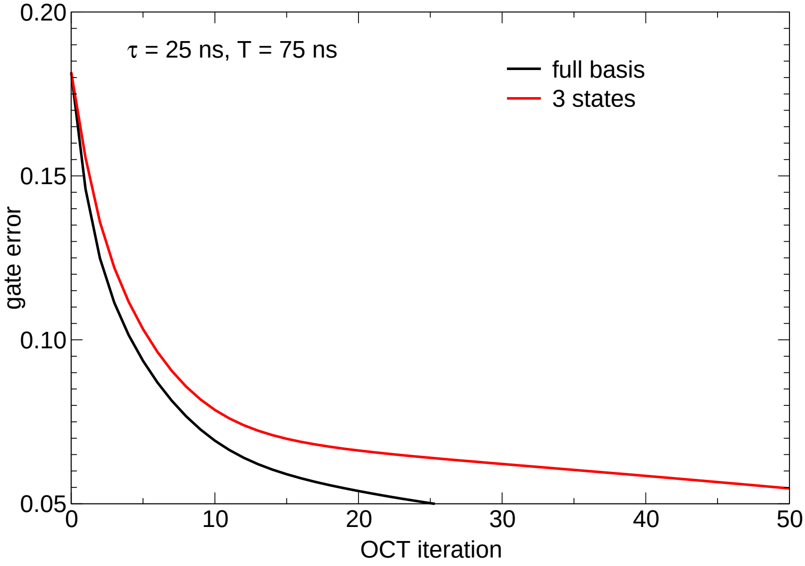
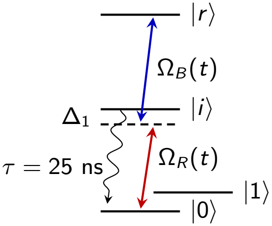


Diagonal gates



no coupling between $|0\rangle$, $|1\rangle$

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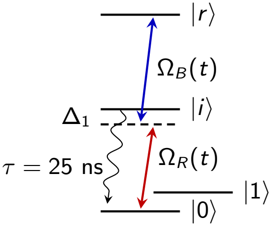

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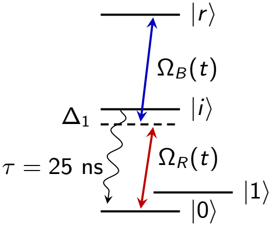

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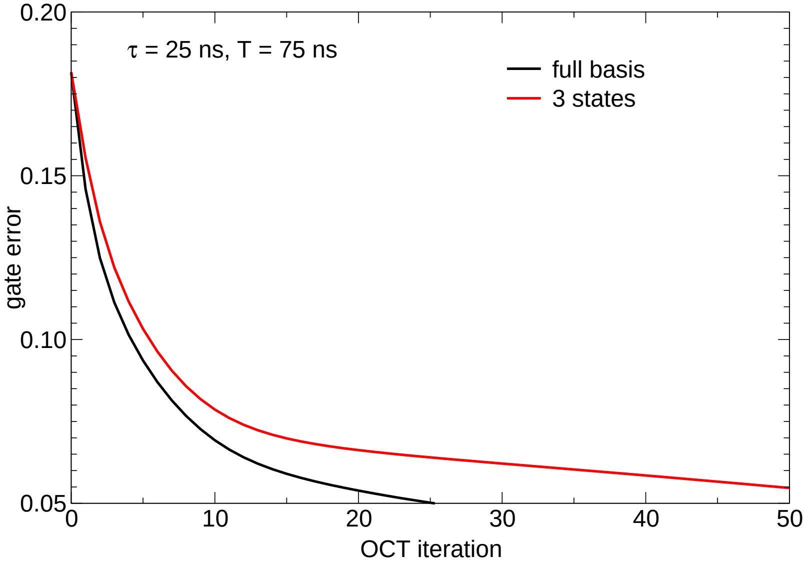

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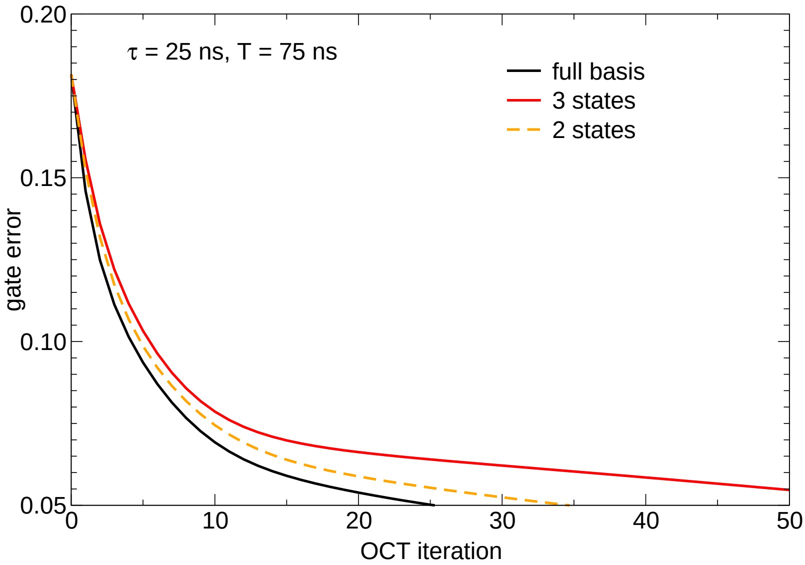

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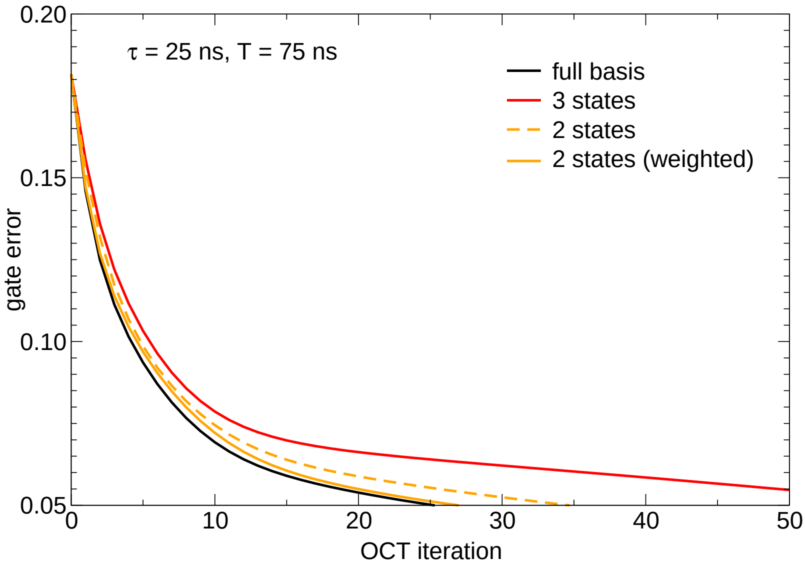
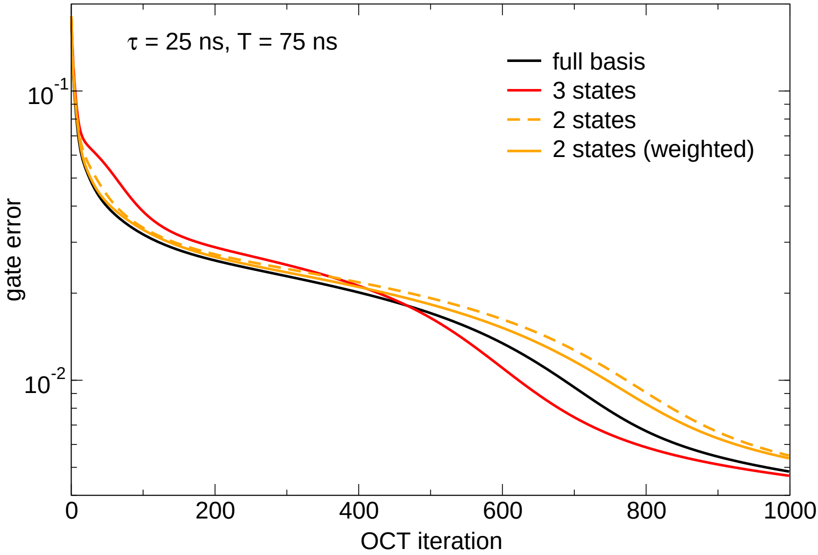


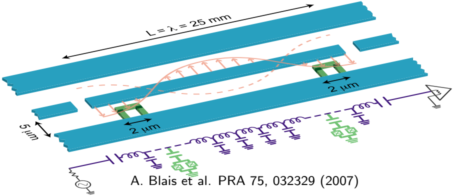
Optimization of a Rydberg gate – asymptotic behavior



Example 2

Optimization of a non-diagonal gate using transmon qubits

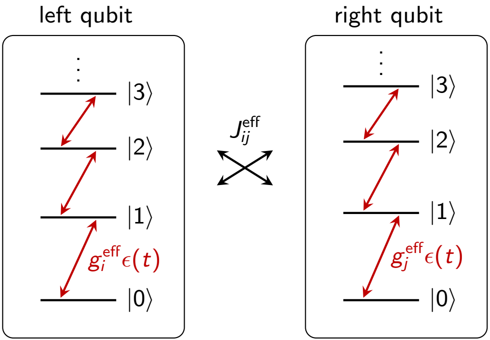
Two coupled transmon qubits



A. Blais et al. PRA 75, 032329 (2007)

Cavity mediates

- driven excitation of qubit
- interaction between left and right qubit



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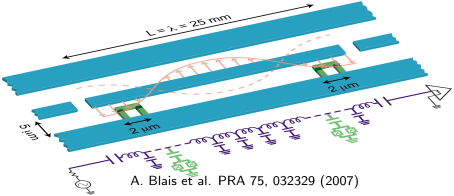

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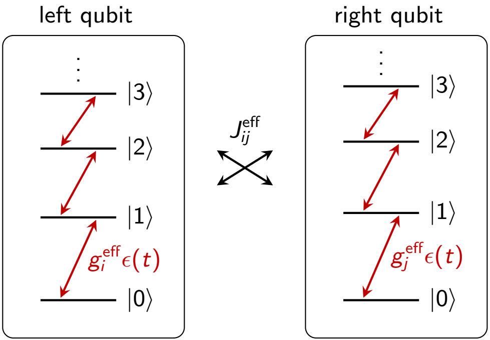
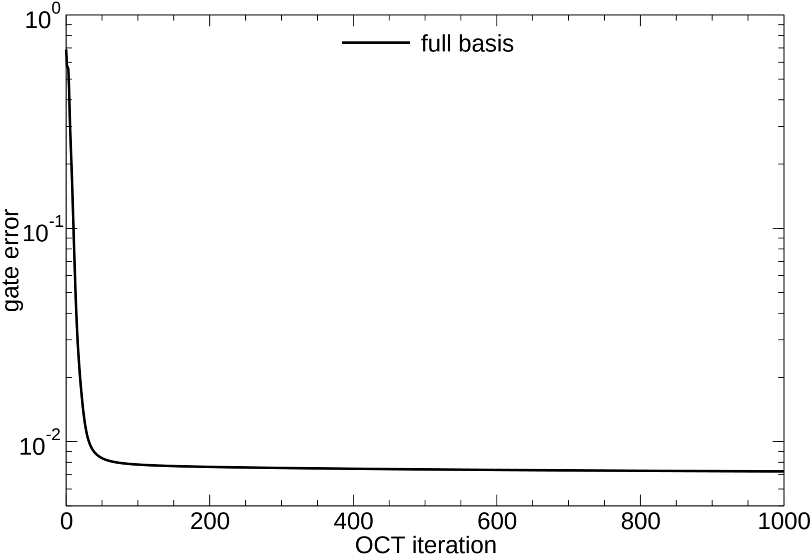


Many gates possible, e.g. $\sqrt{i\text{SWAP}}$:

$$\hat{\mathbf{O}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Optimization of a transmon gate



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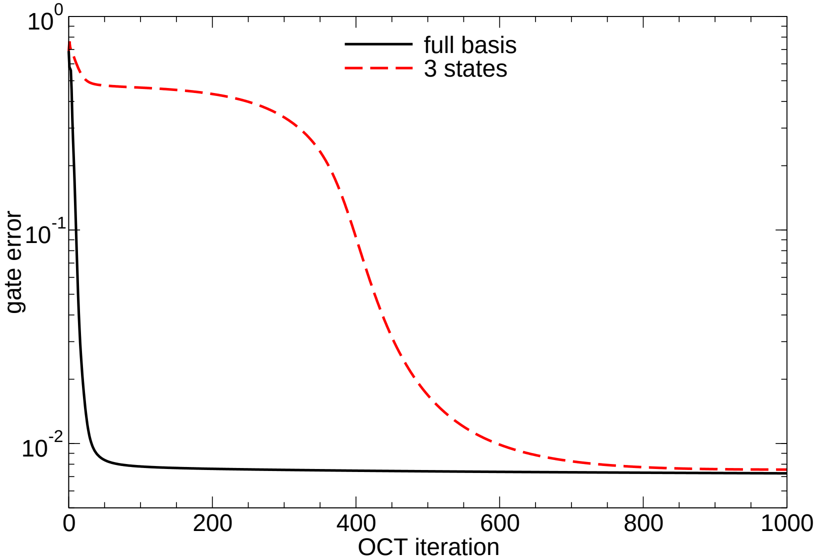

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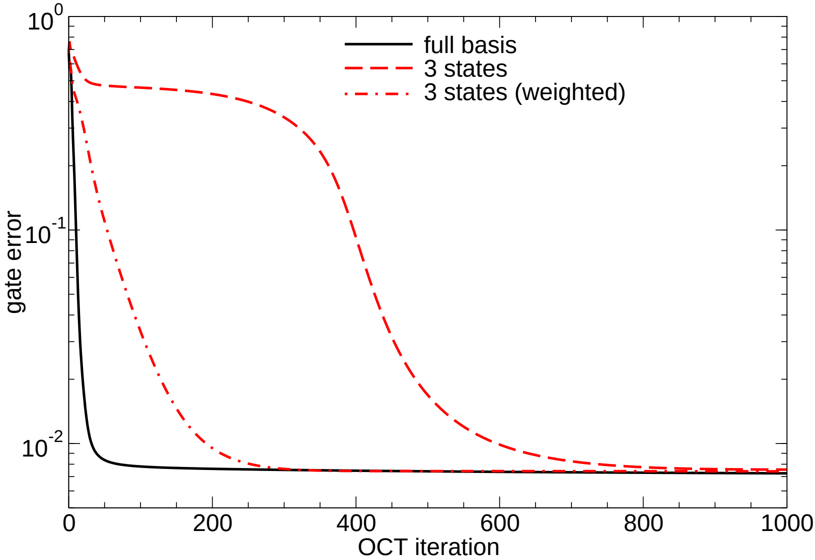
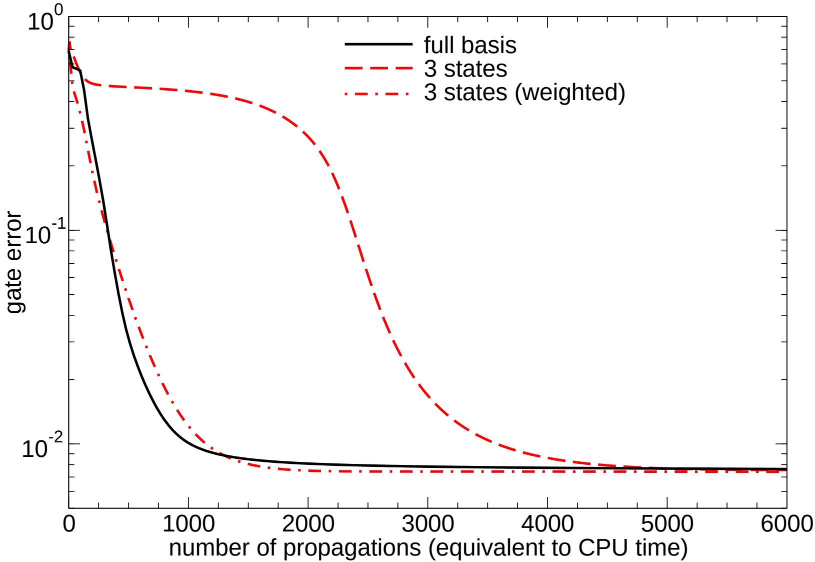


Optimization of a transmon gate – CPU time



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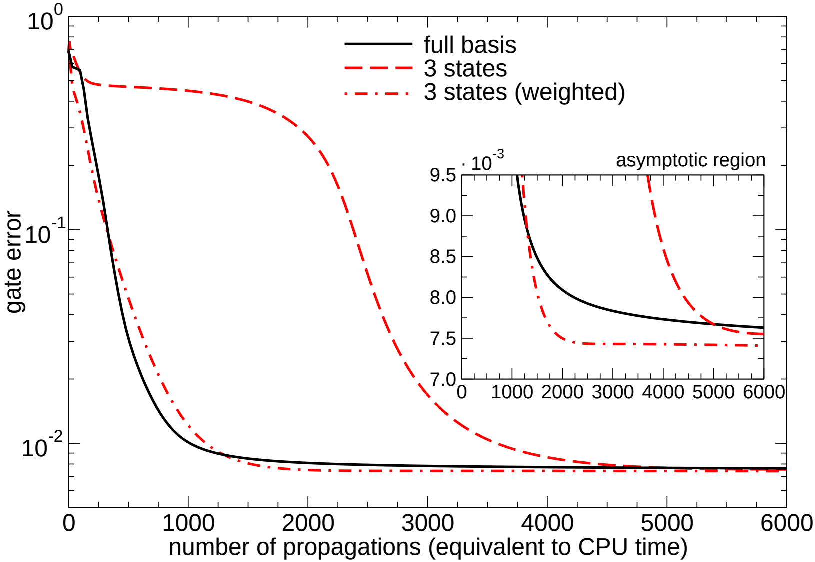

Using pure states only

$$\hat{\rho}_1 = \frac{1}{20}\begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix},\ \hat{\rho}_2 = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix},\ \hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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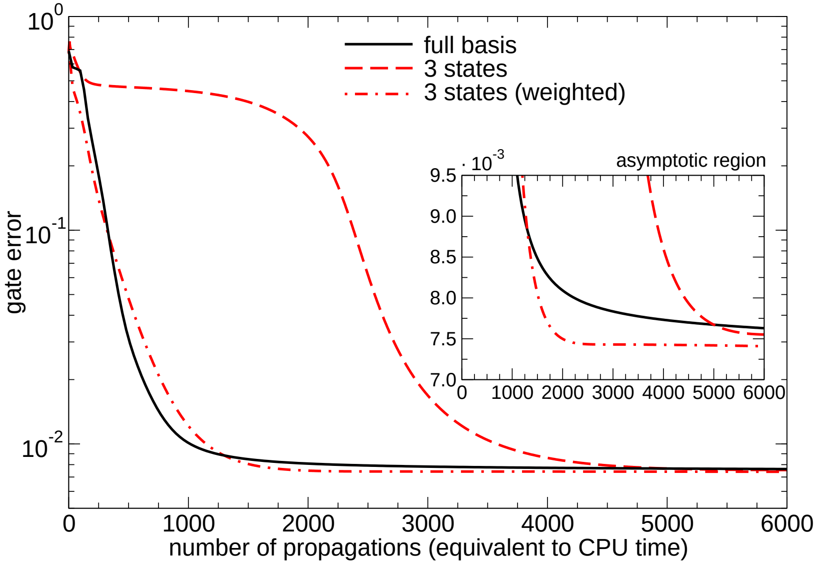
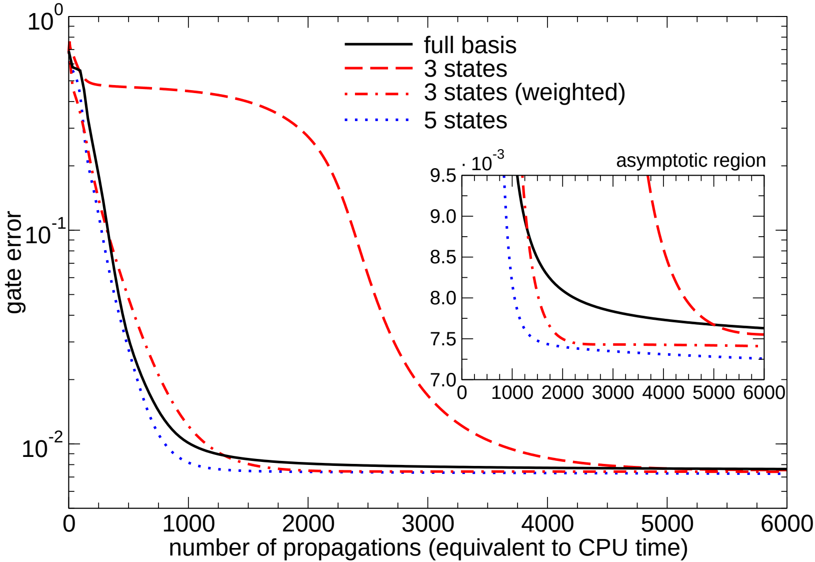
optimization of a transmon gate – CPU time





10^0
10^{-1}
10^{-2}
gate error
0
1000
2000
3000
4000
5000
6000
number of propagations (equivalent to CPU time)
full basis
3 states
3 states (weighted)
5 states
$\cdot 10^{-3}$
asymptotic region
9.5
9.0
8.5
8.0
7.5
7.0
0
1000 2000 3000 4000 5000 6000

Conclusion

- A set of three density matrices is sufficient for gate optimization: (independent of dimension of Hilbert space!)
  - one to check dynamical map on subspace
  - one to check the basis
  - one to check the phases
- Further reduction possible for restricted systems
- States can (should!) be weighted according to physical interpretation

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⇒ Gate optimization in open quantum systems with large Hilbert spaces have become significantly more feasible.

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In press: New Journal of Physics (special issue)

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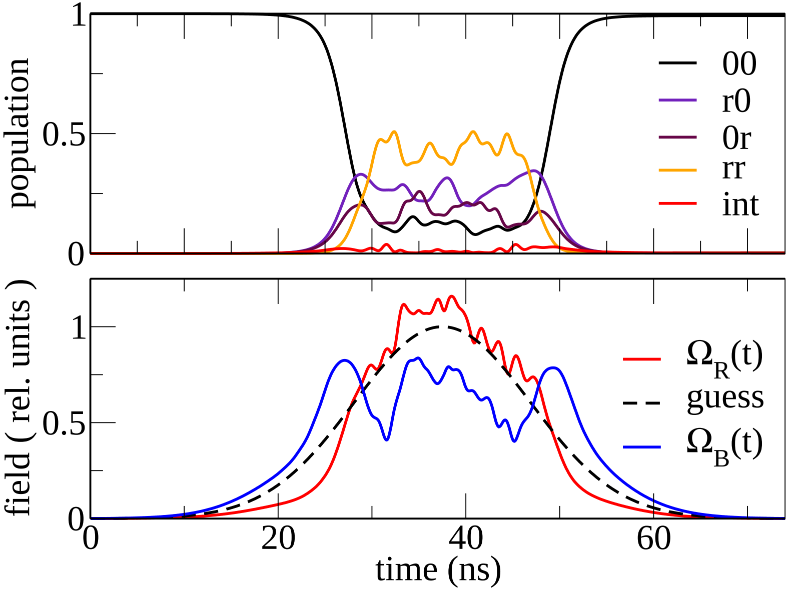
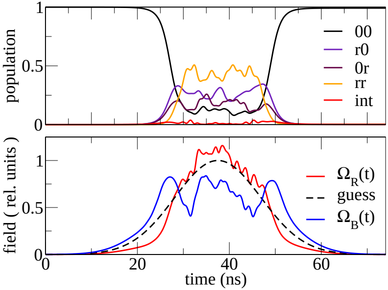
Thank you

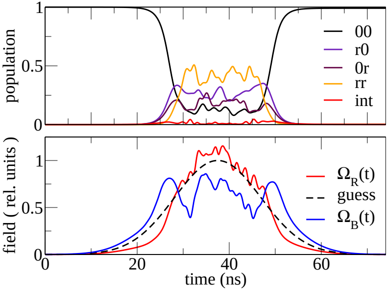
Optimized dynamics of the Rydberg gate



with dissipation, full basis



with dissipation, two states (weighted)



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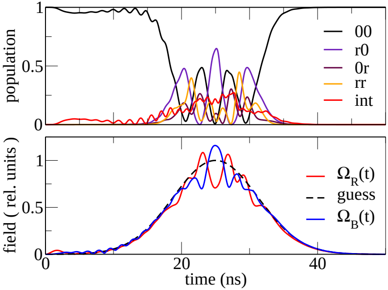

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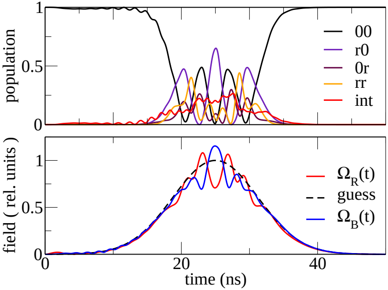
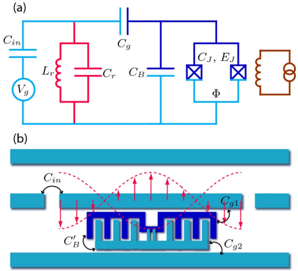
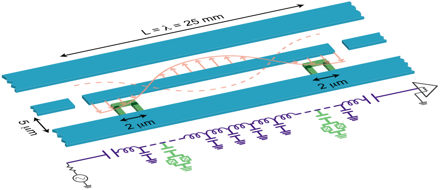


Two Coupled Transmon Qubits



J. Koch et al. PRA 76, 042319 (2007)



A. Blais et al. PRA 75, 032329 (2007)

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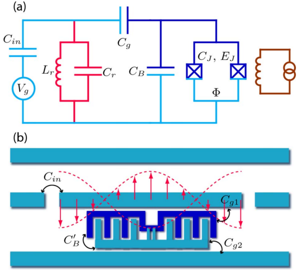

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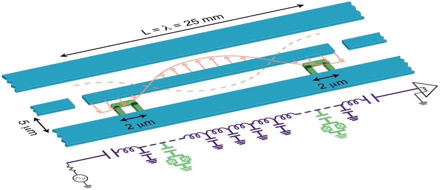

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Full Hamiltonian

$$\hat{\mathbf{H}} = \underbrace{\omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}_{①} + \underbrace{\omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2}_{②} - \underbrace{\frac{1}{2}\left(\alpha_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \alpha_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2\right)}_{③} +$$

$$+ \underbrace{g_1(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2(\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger)}_{④} + \underbrace{\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger}_{⑤}$$

Effective Hamiltonian

$$\hat{\mathsf{H}}_{\text{eff}} = \sum_{q=1,2} \sum_{i=0}^{N_q-1} (\omega_i^{(q)} + \chi_i^{(q)}) \hat{\mathsf{\Pi}}_i^{(q)} + \sum_{q=1,2} \sum_{i=0}^{N_q-1} g_i^{\text{eff}\,(q)} \epsilon(t) (\hat{\mathsf{C}}_i^{+\,(q)} + \hat{\mathsf{C}}_i^{-\,(q)})$$
$$+ \sum_{ij} J_{ij}^{\text{eff}} (\hat{\mathsf{C}}_i^{-\,(1)} \hat{\mathsf{C}}_j^{+\,(2)} + \hat{\mathsf{C}}_i^{+\,(1)} \hat{\mathsf{C}}_j^{-\,(2)}) \,.$$

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with

- $\omega_i^{(q)} = i\omega_q - \frac{1}{2}(i^2 - i)\alpha_q, \qquad g_i^{(q)} = \sqrt{i} g_q$
- $\hat{\mathbf{\Pi}}_i^{(q)} = \left|i\right\rangle \left\langle i\right|_q$, $\hat{\mathbf{C}}_i^{+\,(q)} = \left|i\right\rangle \left\langle i-1\right|_q$
- $\chi_i^{(q)} = \frac{(g_i^{(q)})^2}{(\omega_i^{(q)} - \omega_{i-1}^{(q)} - \omega_c)}$
- $g_i^{\text{eff}\,(q)} = \frac{g_i^{(q)}}{(\omega_i^{(q)} - \omega_{i-1}^{(q)} - \omega_c)}$
- $J_{ij}^{\text{eff}} = \frac{1}{2} g_i^{\text{eff}\,(1)} g_j^{(2)} + \frac{1}{2} g_j^{\text{eff}\,(2)} g_i^{(1)}$

qubit frequency ω_1	4.3796 GHz
qubit frequency ω_2	4.6137 GHz
drive frequency ω_d	4.4985 GHz
anharmonicity α_1	-239.3 MHz
anharmonicity α_2	-242.8 MHz
effective qubit-qubit coupling J	-2.3 MHz
qubit 1,2 decay time T_1	38.0 μs, 32.0 μs
qubit 1,2 dephasing time T_2^*	29.5 μs, 16.0 μs

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Master Equation

$$\mathcal{L}_D(\hat{\boldsymbol{\rho}}) = \sum_{q=1,2} \left(\gamma_q \sum_{i=1}^{N-1} iD \left[|i-1\rangle\langle i|_q \right] \hat{\boldsymbol{\rho}} + \gamma_{\phi,q} \sum_{i=0}^{N-1} \sqrt{i} D \left[|i\rangle\langle i|_q \right] \hat{\boldsymbol{\rho}} \right) ,$$

with $D\left[\hat{\mathbf{A}}\right] \hat{\boldsymbol{\rho}} = \hat{\mathbf{A}} \hat{\boldsymbol{\rho}} \hat{\mathbf{A}}^\dagger - \frac{1}{2} \left(\hat{\mathbf{A}}^\dagger \hat{\mathbf{A}} \hat{\boldsymbol{\rho}} + \hat{\boldsymbol{\rho}} \hat{\mathbf{A}}^\dagger \hat{\mathbf{A}} \right)$

IBM Qubit – Poletto et al. PRL 109, 240505 (2012)

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- Near resonance of α_1 with $\omega_1 - \omega_2$

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- Near resonance of α_1 with $\omega_1 - \omega_2$
- single frequency drive centered between two qubits

Effective Hamiltonian

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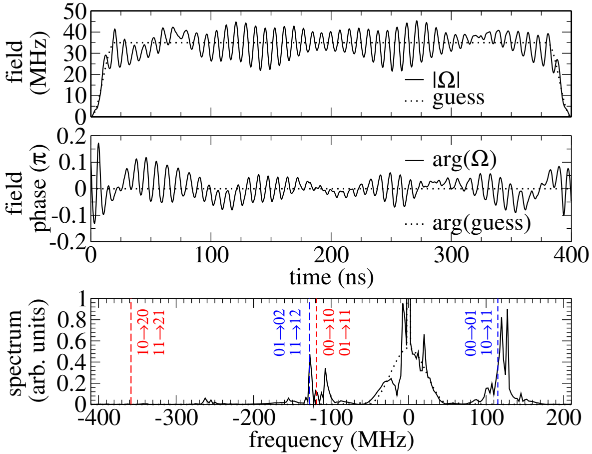
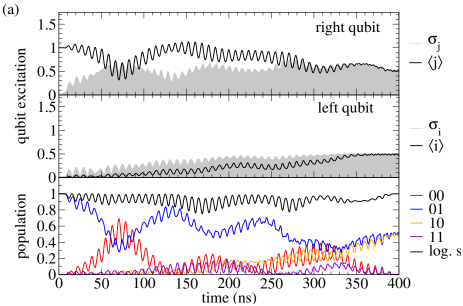
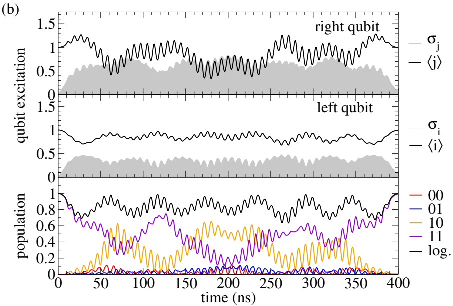
Transmon Optimized Pulse



Transmon Population Dynamics



$\Psi(t=0) = |01\rangle$



$\Psi(t=0) = |11\rangle$