Charting the cQED Design Landscape Using Optimal Control Theory

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Abstract

With recent improvements in coherence times, superconducting transmon qubits have become a promising platform for quantum computing. They can be flexibly engineered over a wide range of parameters, but also require us to identify an efficient operating regime. Using state-of-the-art quantum optimal control techniques, we exhaustively explore the landscape for creation and removal of entanglement over a wide range of design parameters. We identify an optimal operating region outside of the usually considered strongly dispersive regime, where multiple sources of entanglement interfere simultaneously, which we name the quasi-dispersive straddling qutrits (QuaDiSQ) regime. At a chosen point in this region, a universal gate set is realized by applying microwave fields for gate durations of 50 ns, with errors approaching the limit of intrinsic transmon coherence. Our systematic quantum optimal control approach is easily adapted to explore the parameter landscape of other quantum technology platforms.

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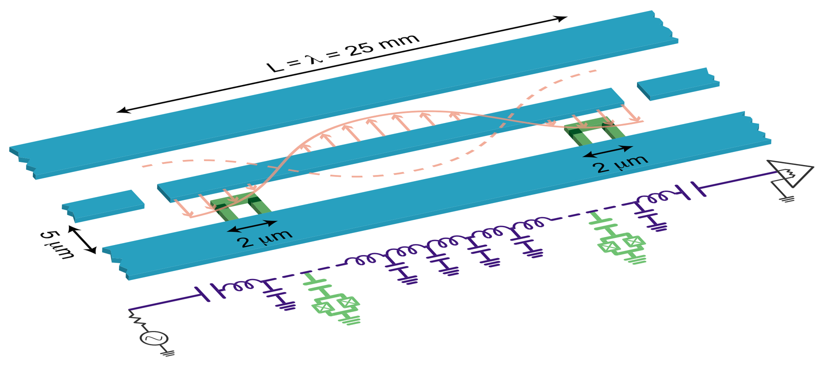
① Two Transmon Qubits Coupled via Cavity



superconducting qubits [1] inside a transmission line resonator, Fig. from [2]

Parameters:

- $\omega_1 = 6.0$ GHz
- $\omega_2 = 5.0 - 7.5$ GHz (vary)
- $\omega_c = 4.5 - 11.0$ GHz (vary)
- $\alpha_1 = -290$ MHz
- $\alpha_2 = -310$ MHz
- $g = 70$ MHz
- $\tau_c = 3.2\ \mu s$ [3];
- $\tau_q = 13.3\ \mu s$ [4]

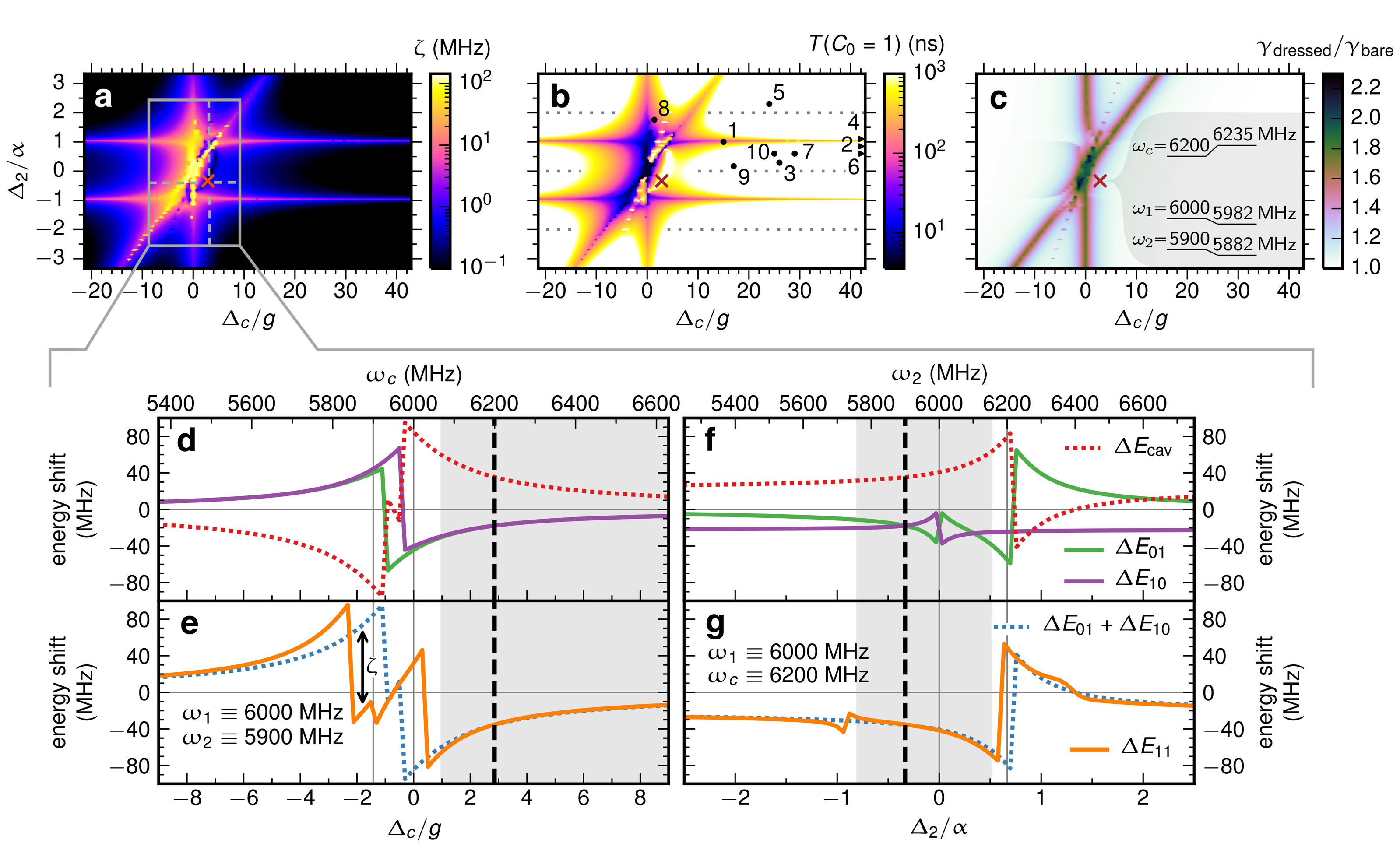
Hamiltonian in the rotating wave approximation ($\delta_q = \omega_q - \omega_d$):

$$\hat{\mathbf{H}} = \hbar\delta_c \hat{\mathbf{a}}^\dagger\hat{\mathbf{a}} + \sum_{q=1,2}\left[\hbar\delta_q\hat{\mathbf{b}}_q^\dagger\hat{\mathbf{b}}_q + \frac{\alpha_q}{2}\hat{\mathbf{b}}_q^\dagger\hat{\mathbf{b}}_q^\dagger\hat{\mathbf{b}}_q\hat{\mathbf{b}}_q + g(\hat{\mathbf{b}}_q^\dagger\hat{\mathbf{a}} + \hat{\mathbf{b}}_q\hat{\mathbf{a}}^\dagger)\right] + \frac{\hbar}{2}\left(\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger\right)$$

avoid treating only dispersive regime by numerical simulation of full Hamiltonian!

relevant parameters to describe landscape: $\Delta_2/\alpha = (\omega_2 - \omega_1)/\alpha$, $\Delta_c/g = (\omega_c - \omega_1)/g$

field-free properties of the Hamiltonian:



field-free entanglement $\zeta = E_{00} + E_{01} + E_{10} - E_{11}$ from interfering dressed energy shifts. ζ varies by order of magnitudes, while dressed decay only varies up to factor 2.3!

② Entanglement creation and destruction

For each point (ω_2, ω_c): find pulse to maximize entanglement (two-qubit gate) *and* pulse to implement local gate $\in SU(2) \otimes SU(2)$, using multi-stage optimization scheme [5].

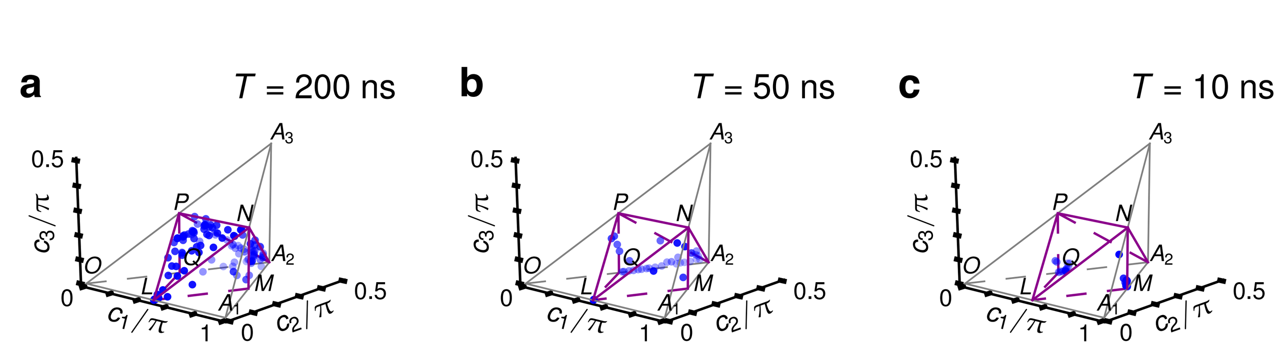
1. Random Search
2. Gradient-free optimization of analytical pulse parameters

Use Nelder-Mead downhill simplex to minimize/maximize entanglement

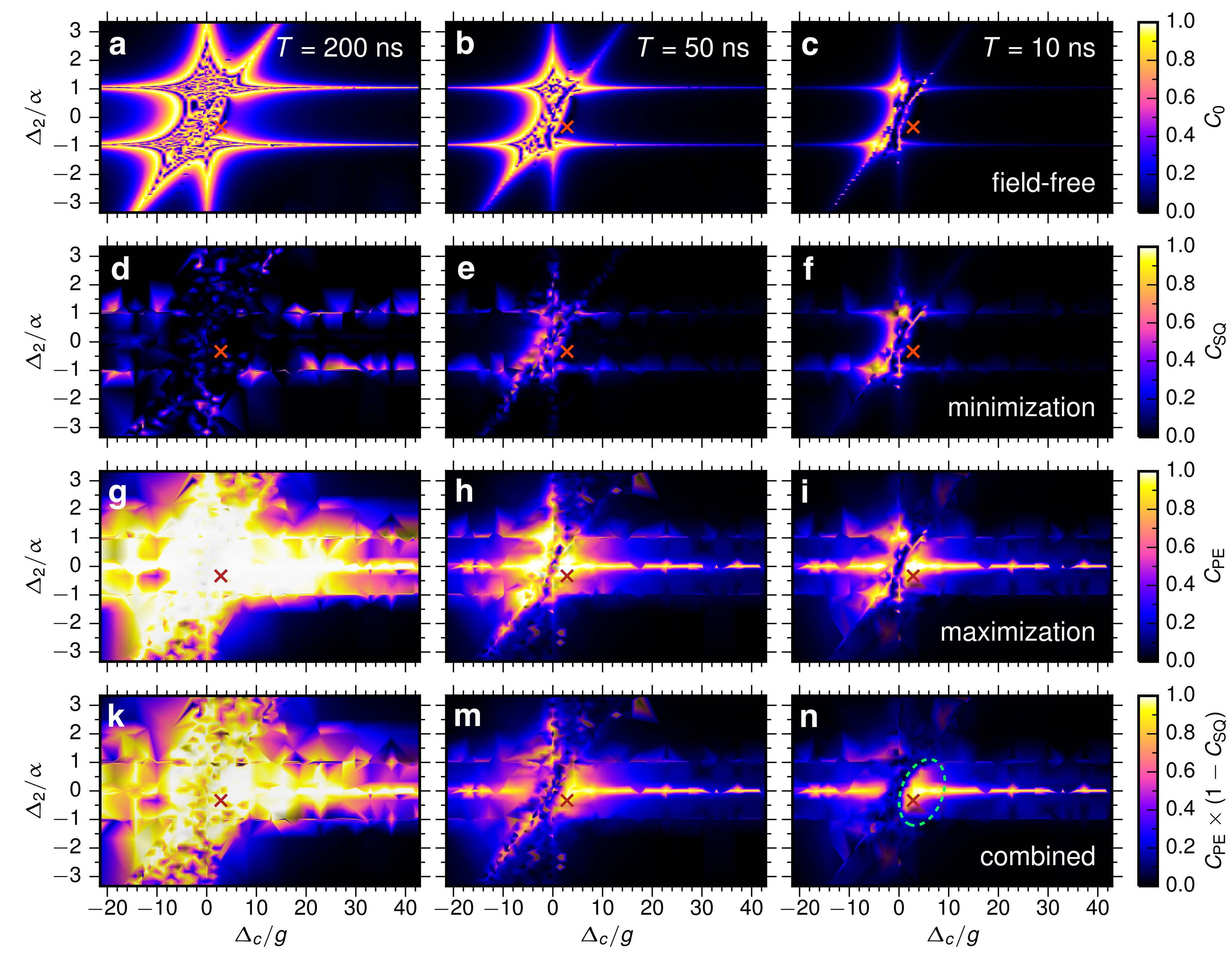
3. Gradient-based optimization (Krotov's method) for fine-tuning

Use Krotov's method [6] to continue optimization of pulse for arbitrary perfect entangler [7] and arbitrary local gate $\in SU(2) \otimes SU(2)$, based on Cartan decomposition [8]

$$\hat{\mathbf{U}} = \hat{\mathbf{k}}_1 \exp\left[\frac{i}{2}\left(c_1\hat{\boldsymbol{\sigma}}_x\hat{\boldsymbol{\sigma}}_x + c_2\hat{\boldsymbol{\sigma}}_y\hat{\boldsymbol{\sigma}}_y + c_3\hat{\boldsymbol{\sigma}}_z\hat{\boldsymbol{\sigma}}_z\right)\right]\hat{\mathbf{k}}_2; \qquad \hat{\mathbf{k}}_{1,2} \in SU(2) \otimes SU(2)$$



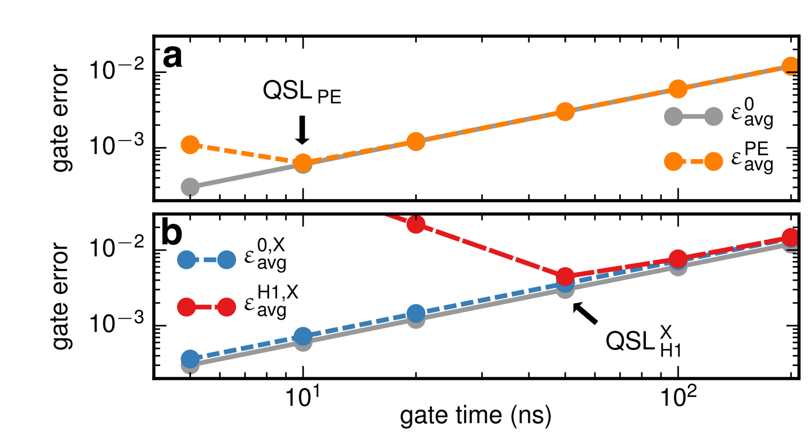
optimization result for maximizing/minimizing entanglement:



optimal point for universal gate implementation in quasi-dispersive "straddling qutrit" regime **("QuaDiSQ")**

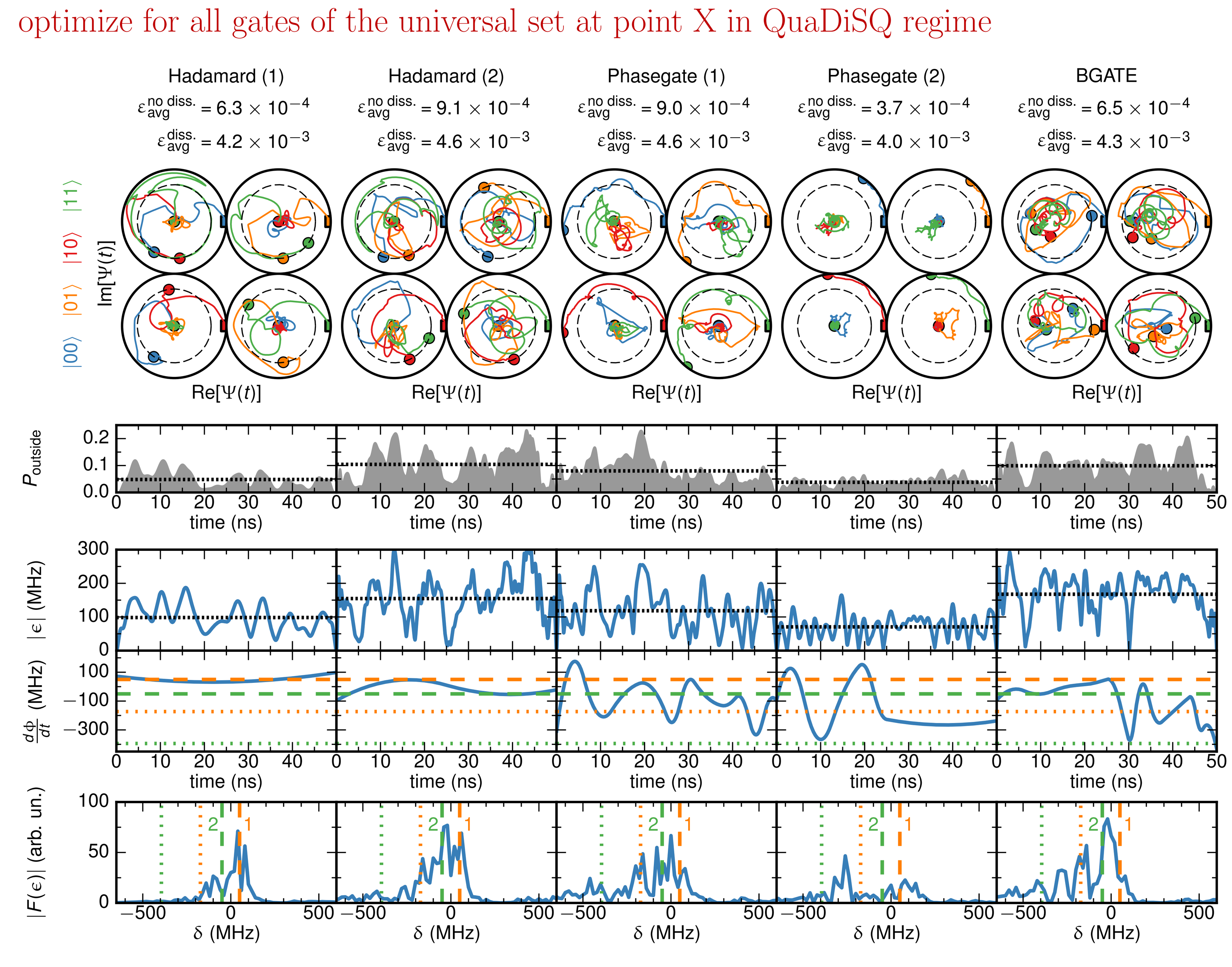
quantum speed limit (QSL):

- decay of qubit imposes limit on the lowest reachable gate error
- QSL at 10 ns when perfect entangler can no longer be realized at minimum error
- at point X: QSL for universal set at 50 ns



③ A universal set of gates

optimize for all gates of the universal set at point X in QuaDiSQ regime



For longer gate duration ($T = 100$ ns) spectral width can be constrained to ± 200 MHz, and pulses can be made robust w.r.t. 1% fluctuation in pulse amplitude (see preprint).

References

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