Minimal Set of States for Optimizing Quantum Gates in Open Quantum Systems

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Universität Kassel

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"Optimal Control of Quantum Systems"

Standard approach to quantum gate optimization

$$\text{CPHASE} = \text{diag}(-1, 1, 1, 1)$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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Goal: Maximize

$$F = \frac{1}{d} \sum_{i=1}^{d} \mathfrak{Re} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^{\dagger} \hat{\mathbf{P}} \hat{\mathbf{U}}(T, 0, \epsilon) \hat{\mathbf{P}} \middle| \Psi_i \right\rangle$$

Two-qubit gates: $d = 4$

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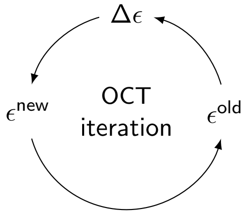
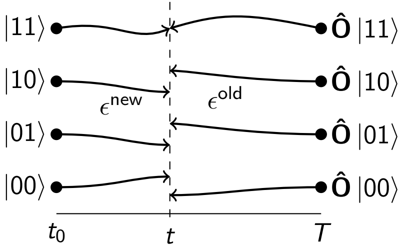


$$\Delta\epsilon(t) \propto \left\langle \chi(t) \middle| \partial_\epsilon \hat{\mathbf{H}} \middle| \Psi(t) \right\rangle$$



In the real world: decoherence

OCT for open quantum systems

$$\hat{\rho}(T) = \mathcal{D}(\hat{\rho}(0)); \qquad \text{e.g. } \frac{\partial \hat{\rho}}{\partial t} = \frac{i}{\hbar}[\hat{\mathbf{H}}, \hat{\rho}] + \mathcal{L}_D(\hat{\rho})$$

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Lift $F = \frac{1}{d} \sum_{i=1}^{d} \mathfrak{Re} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^{\dagger} \hat{\mathbf{P}} \hat{\mathbf{U}}(T, 0, \epsilon) \hat{\mathbf{P}} \middle| \Psi_i \right\rangle$ to Liouville space.

Kallush & Kosloff, Phys. Rev. A 73, 032324 (2006),
...
Schulte-Herbrüggen et al., J. Phys. B 44, 154013 (2011)

$$\Rightarrow F = \frac{1}{d^2} \sum_{j=1}^{d^2} \operatorname{tr}\left[\hat{\mathbf{O}} \hat{\rho}_j(0) \hat{\mathbf{O}}^{\dagger} \hat{\rho}_j(T) \right]$$

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$$\hat{\rho}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\rho}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{\rho}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \ldots$$

d^2 matrices to propagate! (16 for two-qubit gate)

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Claim

We only need to propagate **three** matrices (independent of d), instead of d^2.

A reduced set of density matrices

No need to characterize the full dynamical map!

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① **Do we stay in the logical subspace?**

② **Are we unitary, and if yes, did we implement the right gate?**

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E.g. $\hat{\mathbf{O}} = \text{diag}(-1, 1, 1, 1)$;
For $\hat{\mathbf{U}} = \mathbb{1}$
using just $\hat{\rho}_1$ will not distinguish $\hat{\mathbf{U}}$ from $\hat{\mathbf{O}}$. ($\hat{\mathbf{U}}\hat{\rho}_1\hat{\mathbf{U}}^\dagger = \hat{\mathbf{O}}\hat{\rho}_1\hat{\mathbf{O}}^\dagger$)

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$\hat{\rho}_1$, $\hat{\rho}_2$, $\hat{\rho}_3$ together guarantee that $\mathcal{D}(\hat{\rho})$ is unitary on the logical subspace.

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- $\hat{\rho}_3 = \frac{1}{4}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$

  dynamical map in the logical subspace

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  gate is diagonal in the same basis as $\hat{\mathbf{O}}$

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  Totally rotated state: relative phases between mapped logical eigenstates

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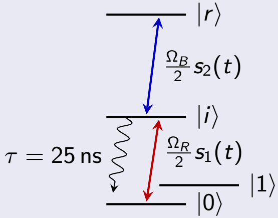
  dynamical map in the logical subspace

An example: Optimization of a Rydberg Gate

Two trapped neutral atoms

Single-qubit Hamiltonian



$$\hat{\mathbf{H}}_{1q} = \begin{pmatrix} 0 & 0 & \frac{\Omega_R}{2} s_1(t) & 0 \\ 0 & E1 & 0 & 0 \\ \frac{\Omega_R}{2} s_1(t) & 0 & \Delta_1 & \frac{\Omega_B}{2} s_2(t) \\ 0 & & \frac{\Omega_B}{2} s_2(t) & \Delta_2 \end{pmatrix}$$

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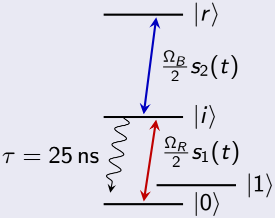

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Two-qubit Hamiltonian

$$\hat{\mathbf{H}}_{2q} = \hat{\mathbf{H}}_{1q} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{H}}_{1q} - U \left| rr \right\rangle \left\langle rr \right|$$

dipole-dipole interaction when both atoms in Rydberg state

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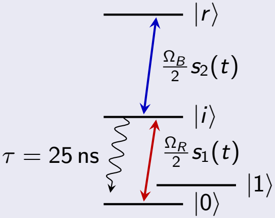

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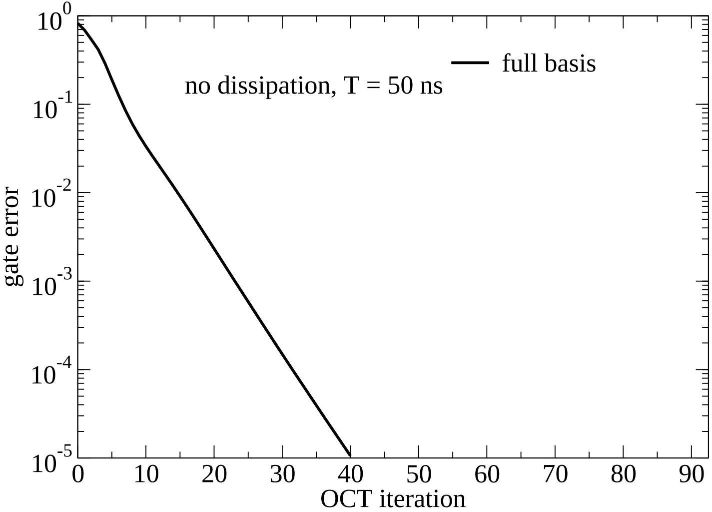
no coupling between $|0\rangle$, $|1\rangle$ $\Rightarrow$ only diagonal gates

$$\hat{\mathbf{U}} = \text{diag}(e^{i\phi_{00}}, e^{i\phi_{01}}, e^{i\phi_{10}}, e^{i\phi_{11}})$$

first: optimize in Liouville space
– but without dissipation

OCT with a reduced set of states. . . without dissipation



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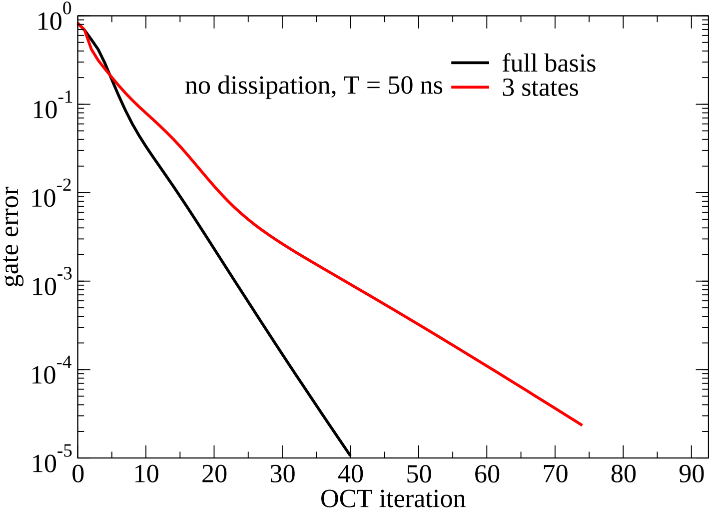
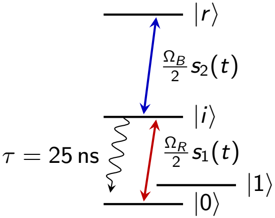


Diagonal Gates



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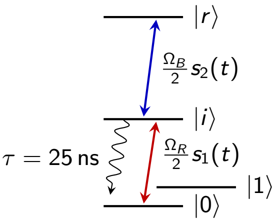

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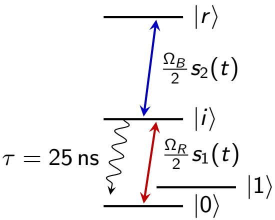

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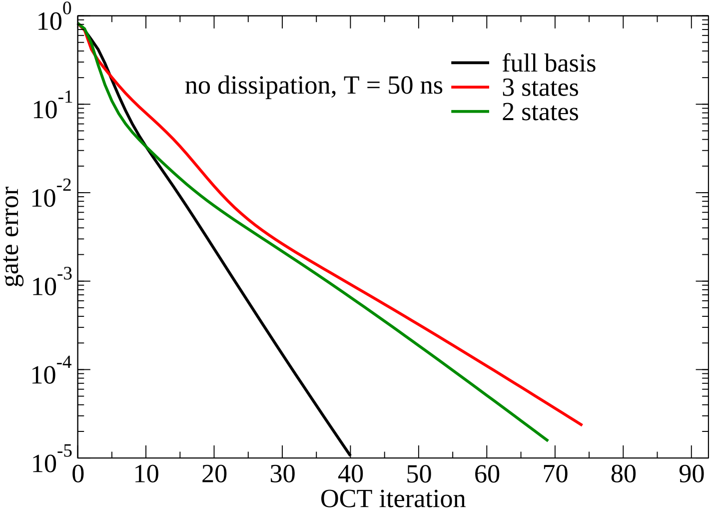
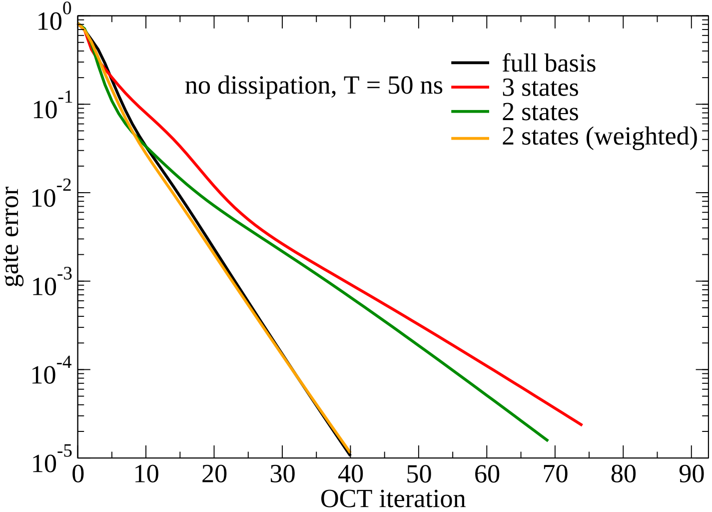
OCT with a reduced set of states. . . without dissipation

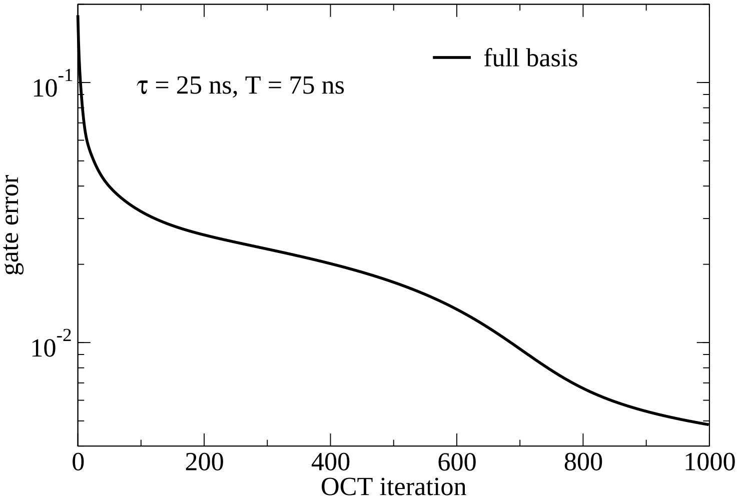


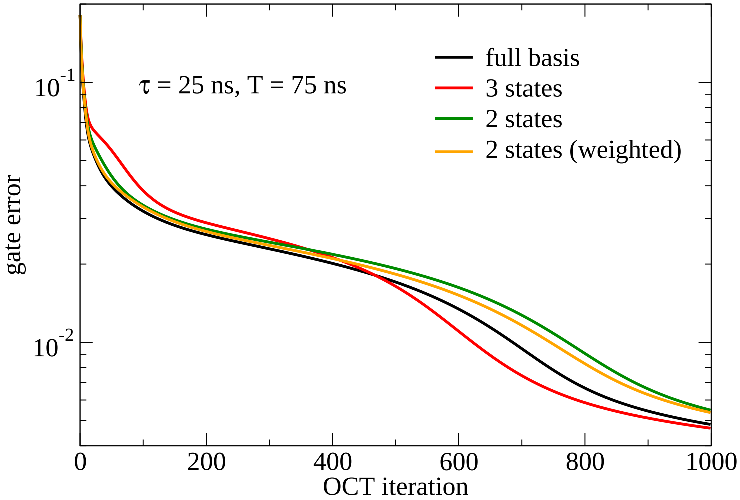


no dissipation, T = 50 ns
full basis
3 states
2 states
2 states (weighted)
gate error
OCT iteration
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10
20
30
40
50
60
70
80
90

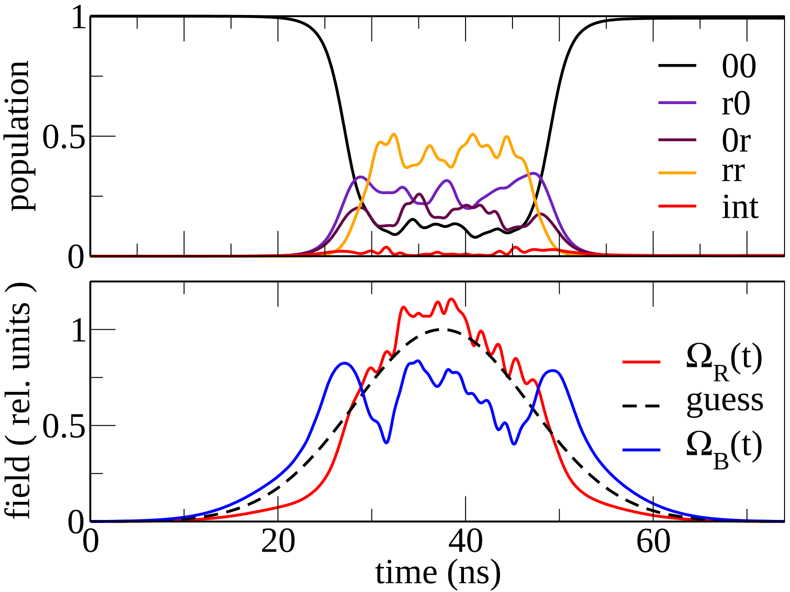
with dissipation

OCT with a reduced set of states. . . with dissipation




τ = 25 ns, T = 75 ns
full basis
3 states
2 states
2 states (weighted)
gate error
10^-1
10^-2
0
200
400
600
800
1000
OCT iteration

Optimized dynamics



Summary and Outlook

- A set of three density matrices is sufficient for gate optimization: (independent of dimension of Hilbert space!)
  - one to check dynamical map on subspace
  - one to check the basis
  - one to check the phases
- Further reduction possible for restricted systems
- States can be weighted according to physical interpretation

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To do:

- Test with more complex Hamiltonians allowing non-diagonal gates (Transmon gates)

Thank You!

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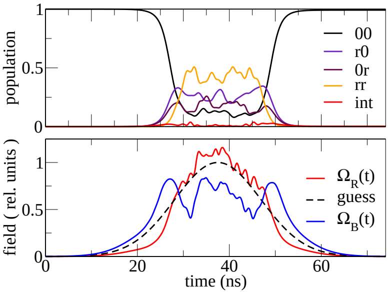
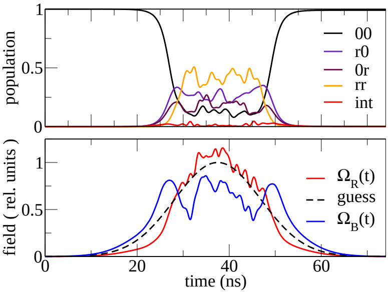
Reich, Gualdi, Koch. arXiv:1305.3222 (2013)

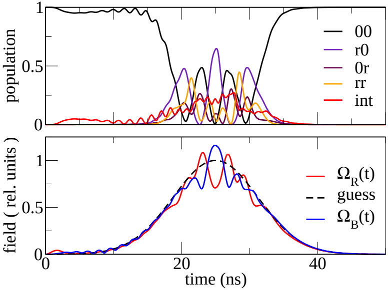
with dissipation, full basis



with dissipation, two states (weighted)



without dissipation, full basis



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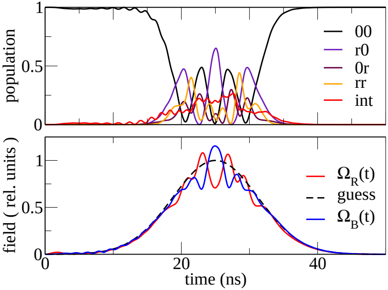

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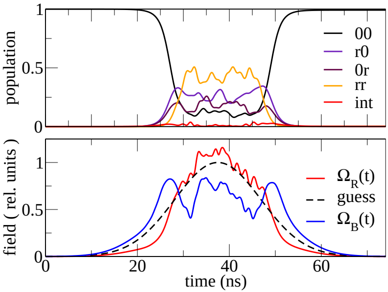

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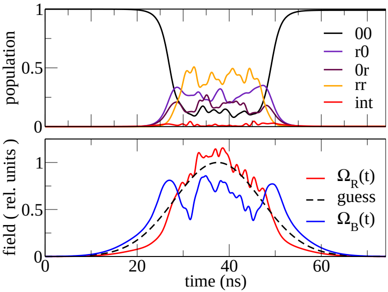
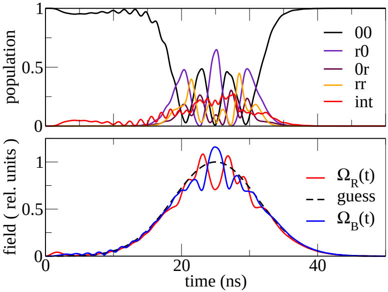


without dissipation, full basis



without dissipation, three states