



Tools of Optimal Control for the Design of Robust Atom Interferometers

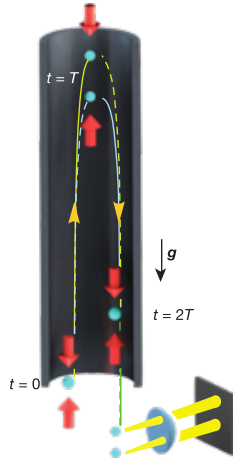
Michael Goerz¹, P. Kunz¹, M. Kasevich², V. Malinovsky¹

¹U.S. Army Research Lab, ²Stanford University

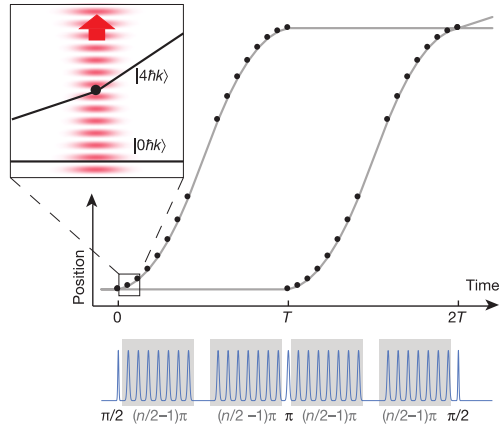
Quantum 2.0

September 2020

10 m atomic fountain at Stanford (Kasevich lab): ultracold ^{87}Rb atomic cloud

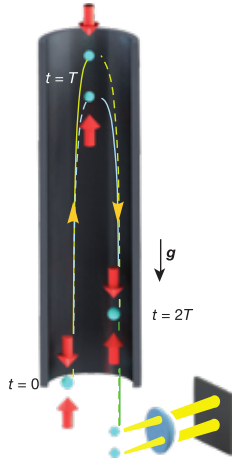


$$\Delta\phi = -2k_{\max}gT^2$$



Kovachi et al. *Nature* **528**, 530 (2015)

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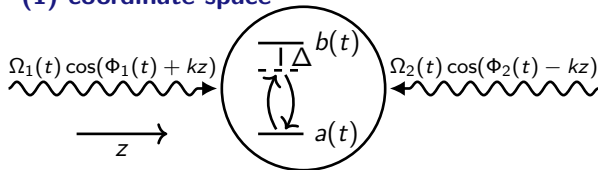


Applications:

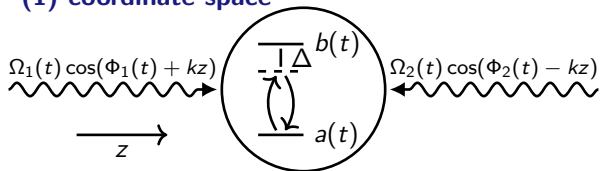
- inertial navigation
- gravitational sensing
- test of equivalence principle

$$\Delta\phi = -2k_{\max}gT^2$$

(1) coordinate space



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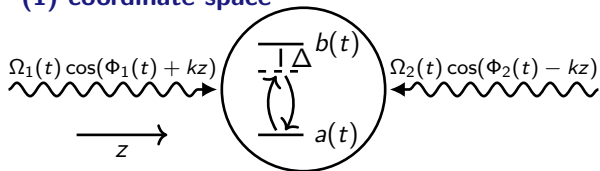


⇓ (2) adiabatic elimination of $b(t)$ — RWA

effective amplitude $\Omega(t) = \Omega_1(t)\Omega_2(t)/4\Delta$

effective phase $\phi(t) = \Phi_1(t) - \Phi_2(t)$

(1) coordinate space

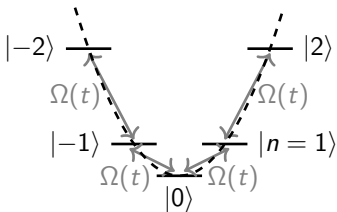


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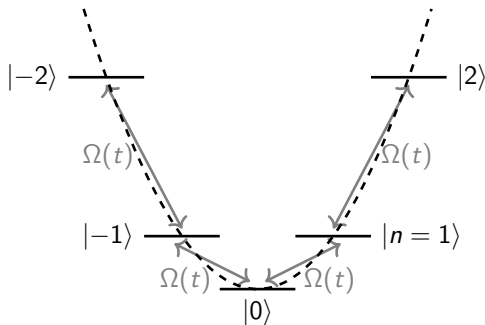
⇓ (3) discretization in momentum space (initial momentum p_0)



$$E_n(t) \approx n^2 \hbar \omega_k + 2n\beta \hbar \omega_k + n \hbar \dot{\phi}(t))$$

$$\omega_k = 2\hbar k^2 / m$$

$$\beta = p_0 / 2\hbar k$$



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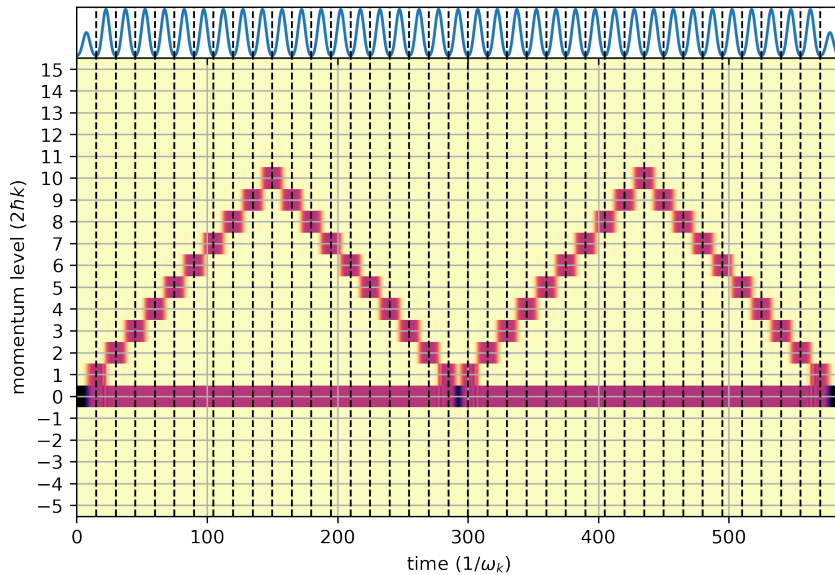
$$\omega_k = 2\hbar k^2 / m$$

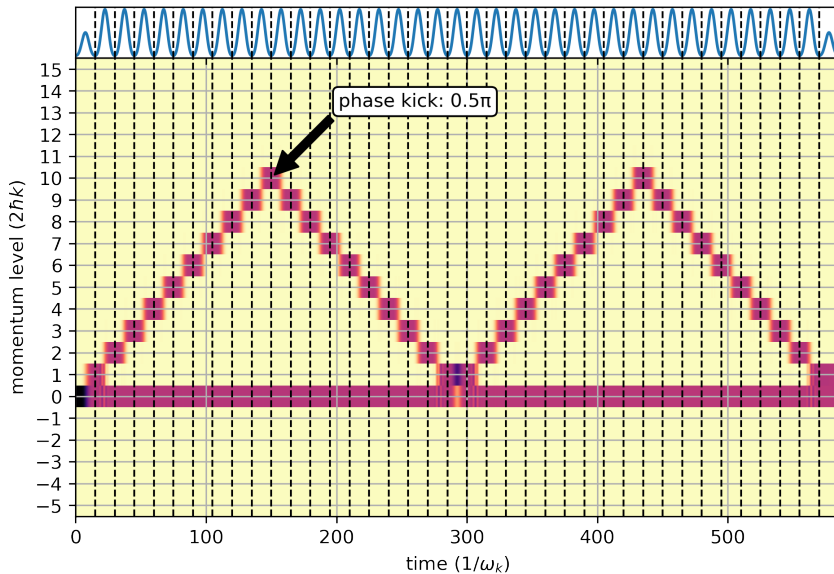
$$\beta = p_0 / 2\hbar k$$

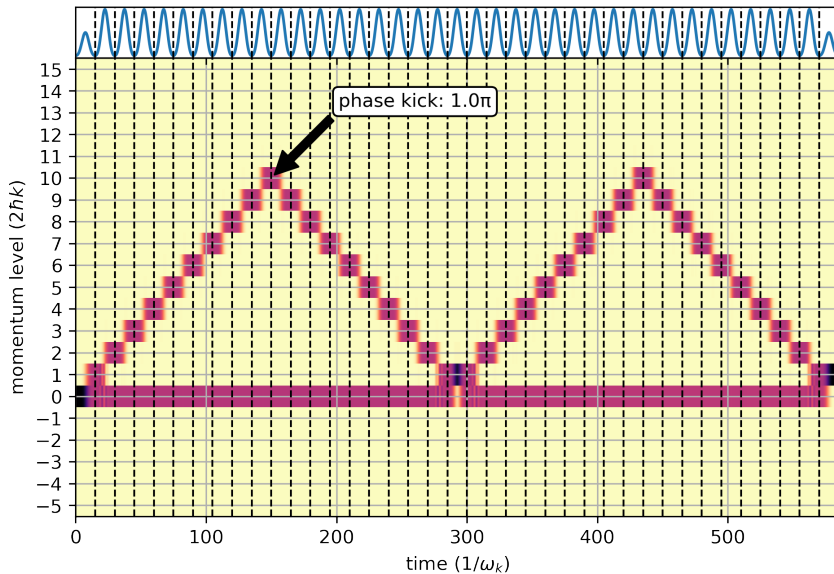
Train of Rabi pulses:

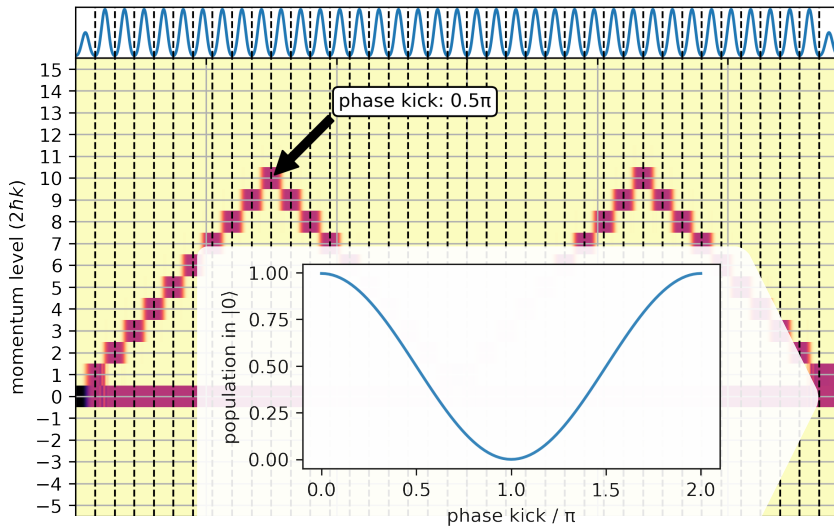
Choose laser frequency such that levels n_0 and $n_0 + 1$ are shifted into resonance.

$$\phi(t) = \omega_L t; \quad \omega_L = -\omega_k(2n_0 + 1)$$









Robustness?

$$\hat{H} = \sum_n \left(E_n^{(0)}(t) + 2n\beta\hbar\omega_k \right) |n\rangle\langle n| - \Omega(t) (|n\rangle\langle n+1| + \text{H.c.})$$

Robustness?

$$\hat{H} = \sum_n \left(E_n^{(0)}(t) + 2n\beta\hbar\omega_k \right) |n\rangle\langle n| - \mu S(t) (|n\rangle\langle n+1| + \text{H.c.})$$

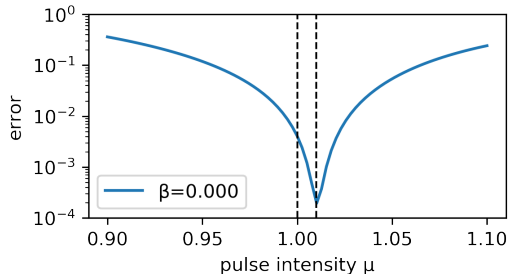
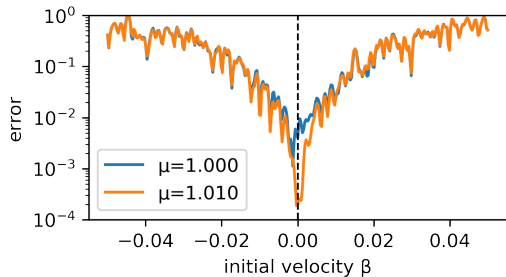
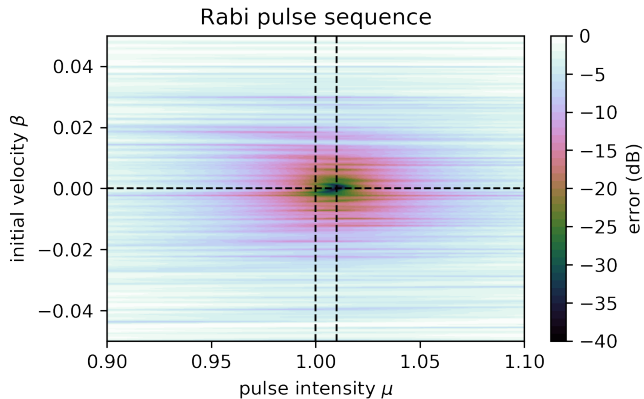
Robustness?

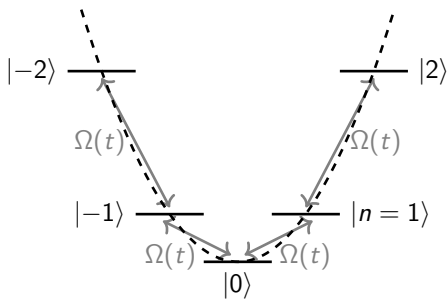
$$\hat{H} = \sum_n (E_n^{(0)}(t) + 2n\beta\hbar\omega_k) |n\rangle\langle n| - \mu S(t) (|n\rangle\langle n+1| + \text{H.c.})$$

noise parameters:

- μ (pulse intensity),
- $\beta = p_0/2\hbar k$ (initial velocity)

Robustness of sequential Rabi pulses





$$E_n(t) \approx n^2 \hbar \omega_k + 2n\beta \hbar \omega_k + n \hbar \dot{\phi}(t)$$

$$\omega_k = 2\hbar k^2 / m$$

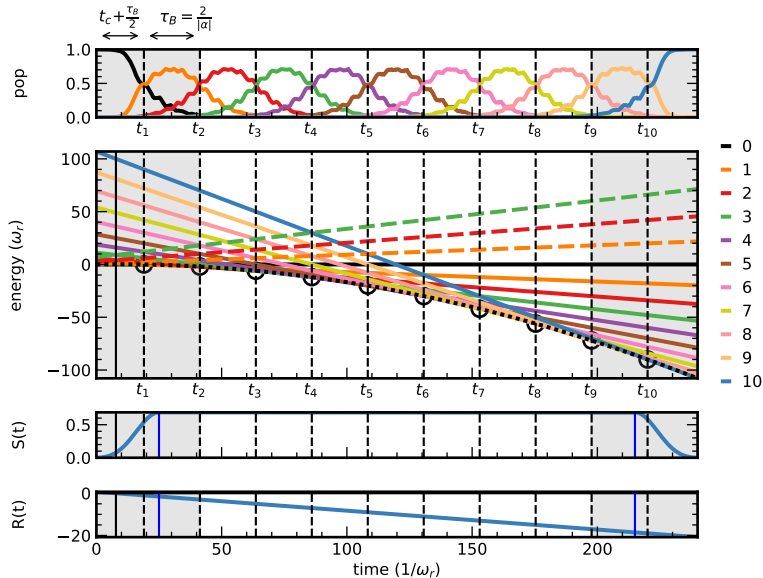
$$\beta = p_0 / 2\hbar k$$

Rapid Adiabatic Passage (RAP)

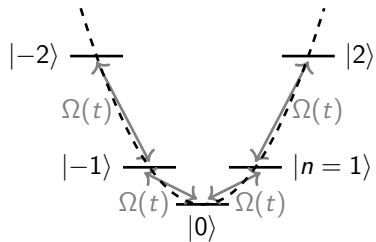
Linear frequency chirp running over transitions

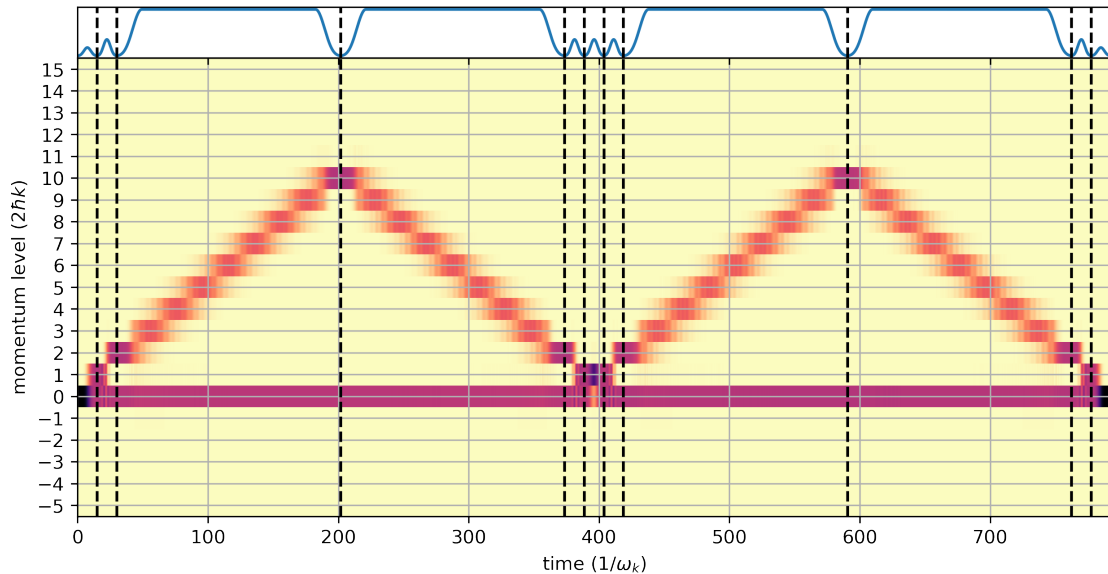
$$\phi(t) = -\frac{1}{2}\alpha\omega_k(t - t_c)^2$$

Malinovsky, Berman. *Phys. Rev. A* **68**, 023610 (2003)

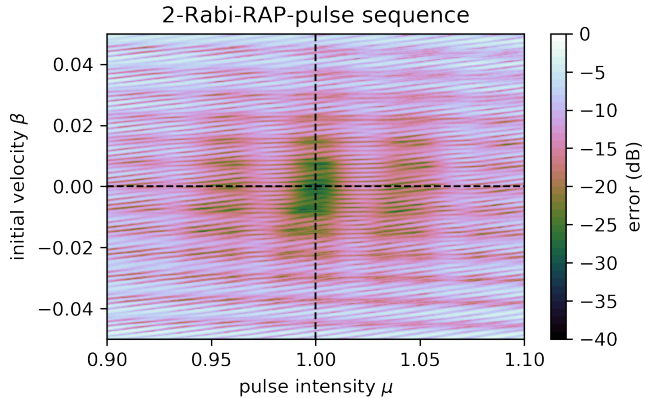


RAP population transfer

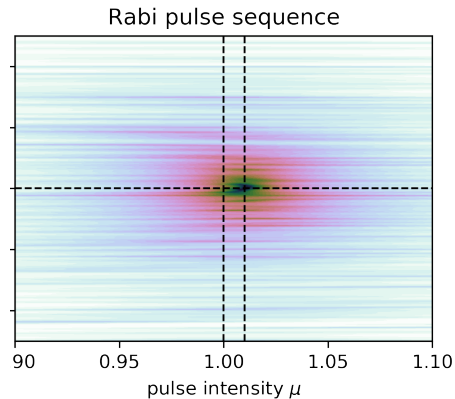
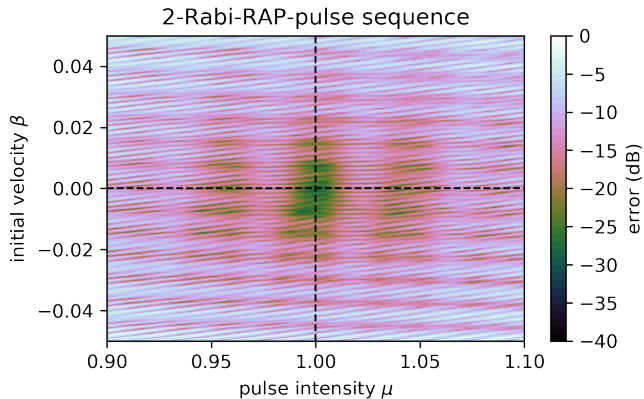




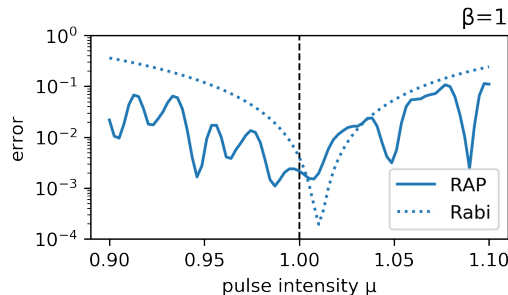
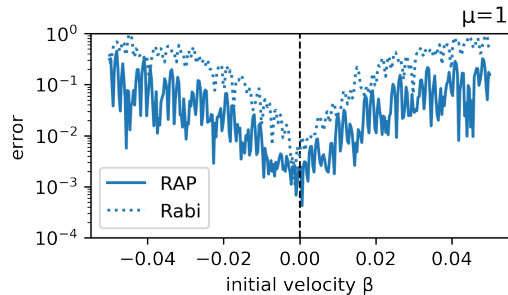
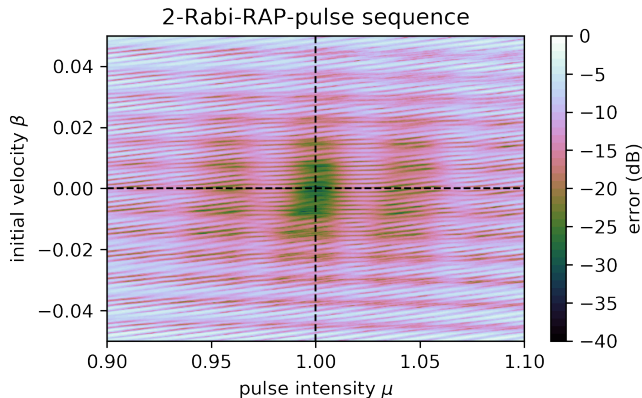
Robustness of RAP pulse scheme



Robustness of RAP pulse scheme



Robustness of RAP pulse scheme



numerical optimal control:

Iteratively minimize figure of merit

$$J_{T,\text{sm}} = \varepsilon_{\text{sm}} = 1 - |\langle \psi^{\text{tgt}} | \psi(T) \rangle|^2;$$

Find $S(t)$ that ensures $|0\rangle \rightarrow |\psi^{\text{tgt}}\rangle = |10\rangle$

Implementation of Krotov's method: github.com/qucontrol/krotov

Goerz et al., *SciPost Phys.* **7**, 80 (2019)

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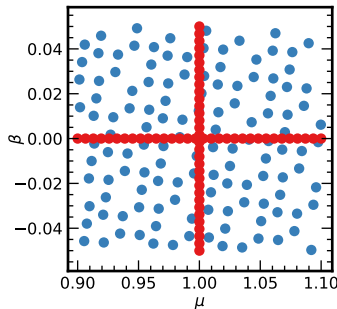
Implementation of Krotov's method: github.com/qucontrol/krotov

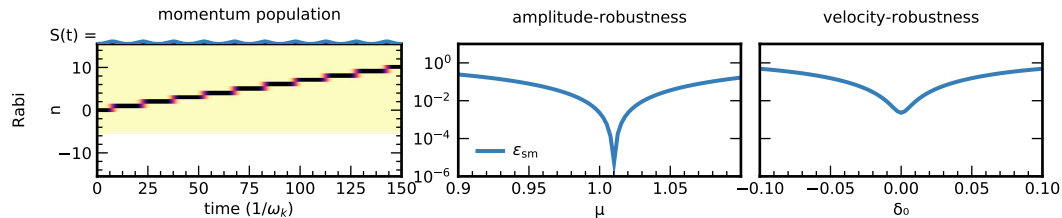
Goerz et al., *SciPost Phys.* **7**, 80 (2019)

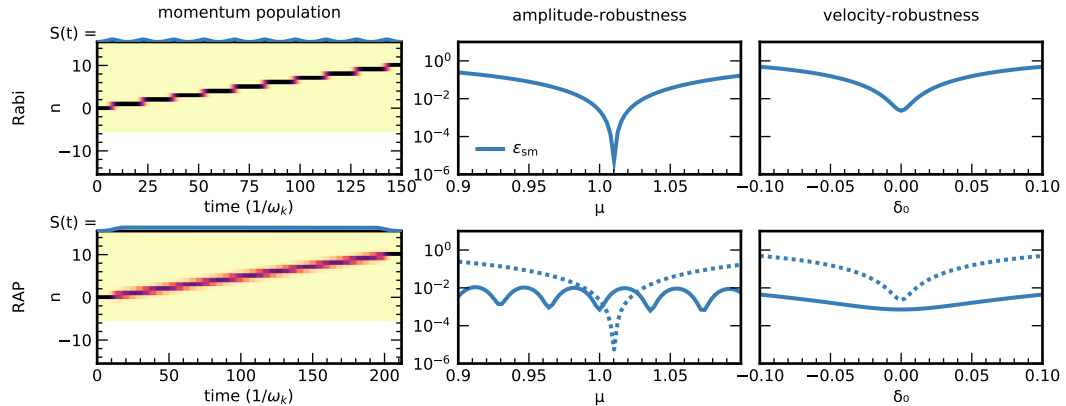
ensemble optimization

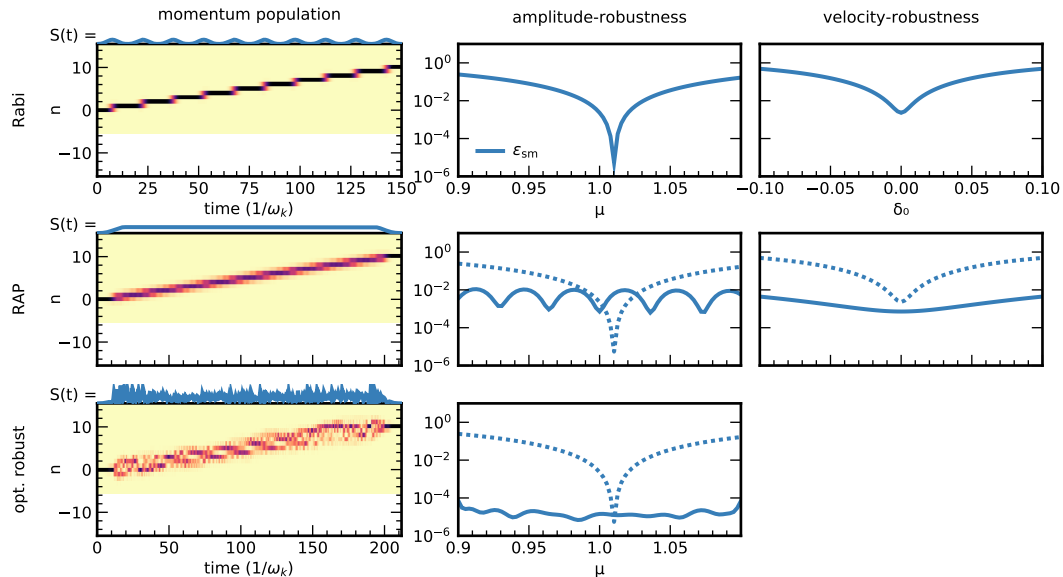
simultaneously optimize over average of random noise realizations

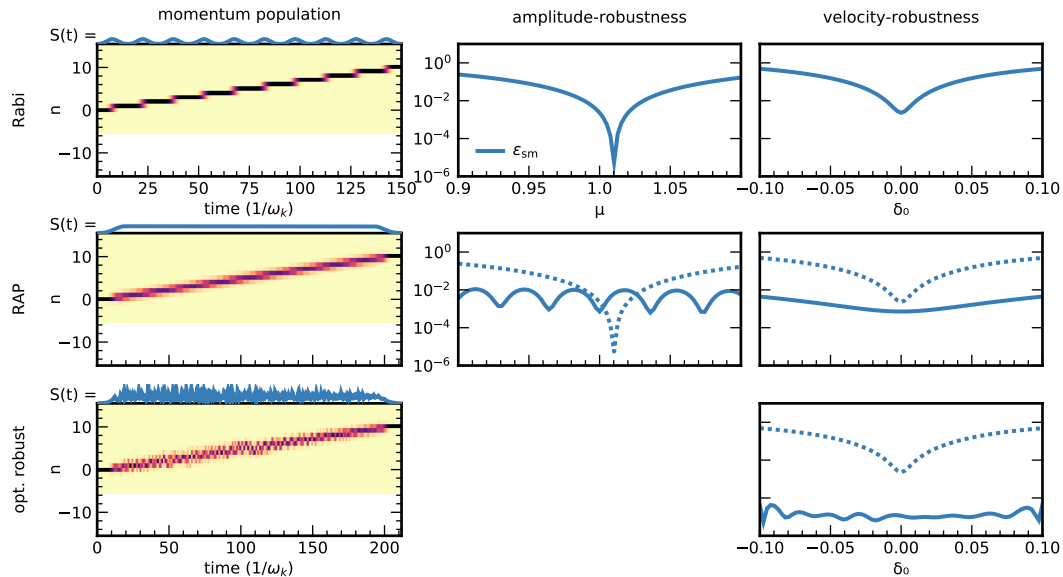
Goerz et al., *Phys. Rev. A* **90**, 032329 (2014)

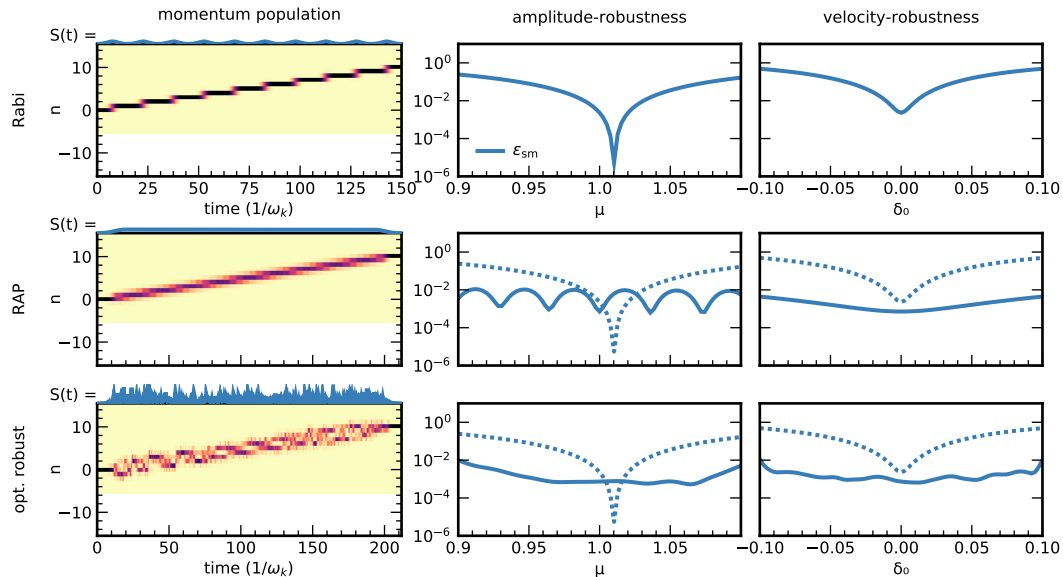












Summary

- Analyzed robustness for train of Rabi pulses
- More robust: Rapid Adiabatic Passage
- Order of magnitude improvement in robustness:
ensemble optimization

Outlook

- Optimized scheme for full interferometer
- Experimental constraints

Thank you!