Designing Quantum Technology with Optimal Control

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Rigetti Computing Seminar
Berkeley

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controllable Hamiltonian:

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_0 + u_1(t)\hat{\mathbf{H}}_1 + u_2(t)\hat{\mathbf{H}}_2 + \dots$$

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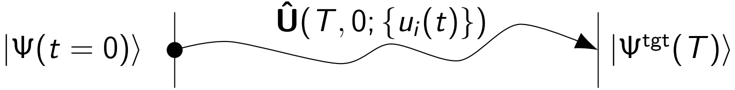

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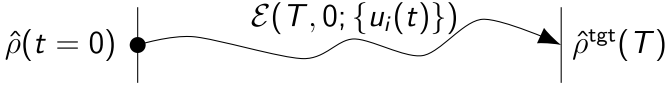

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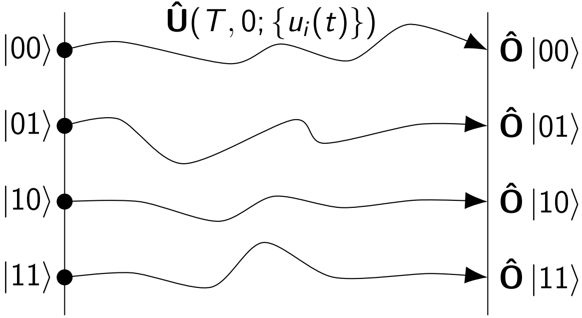
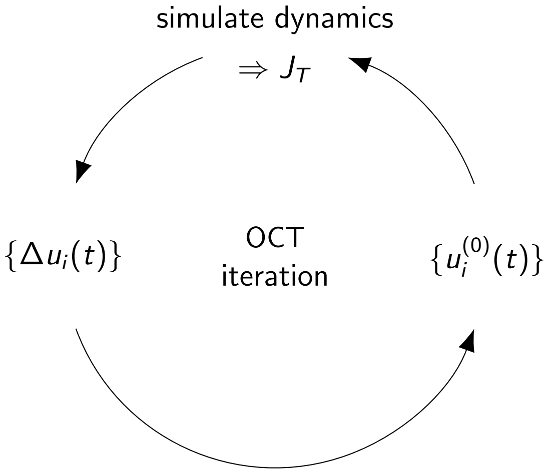


numerical optimal control: iterative improvement



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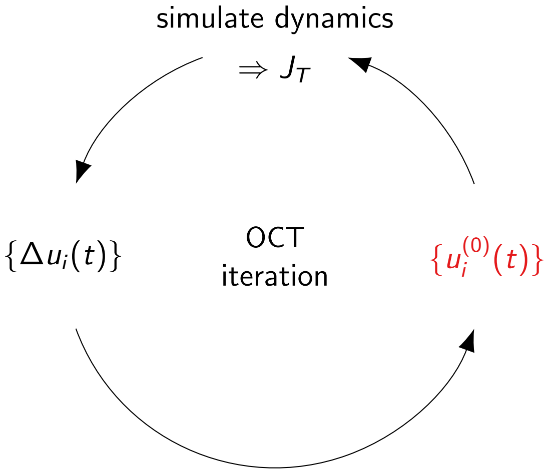

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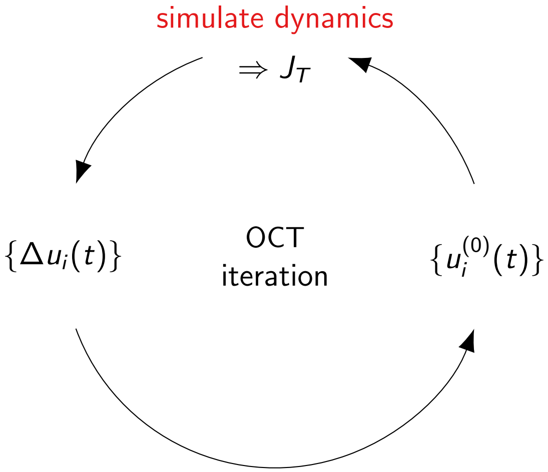

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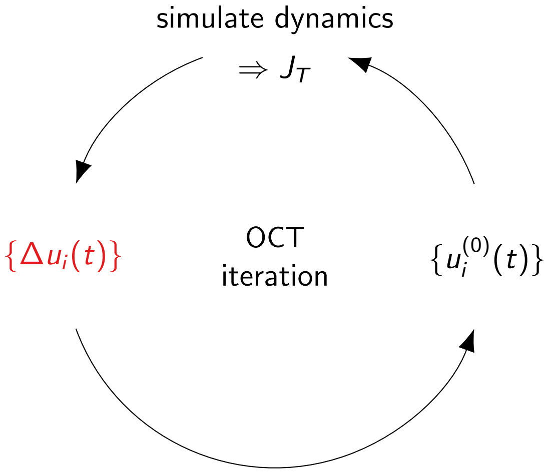

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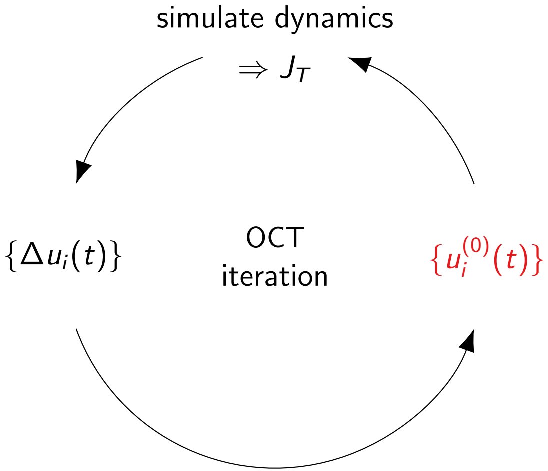

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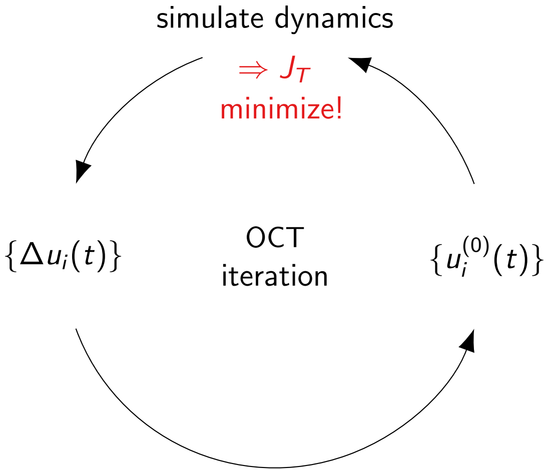

Hamiltonian design

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- quantum speed limit

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- robustness – experimental noise

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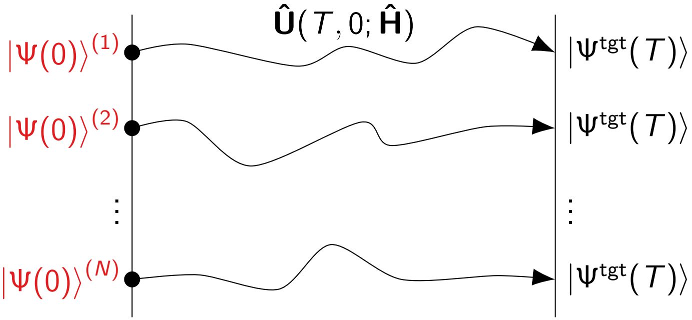

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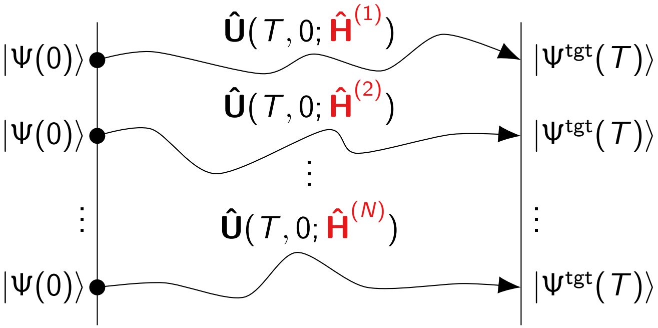

$\Rightarrow$ Goerz et al. Phys. Rev. A 90, 032329 (2014)

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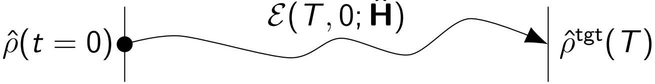

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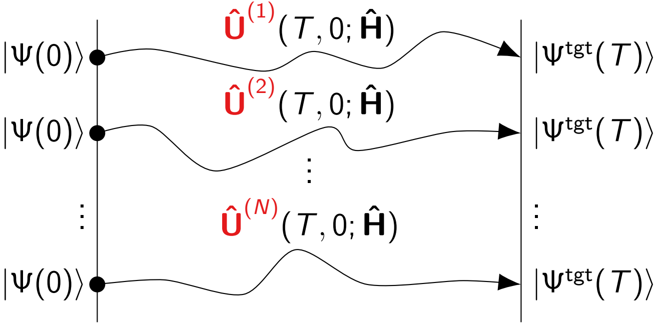

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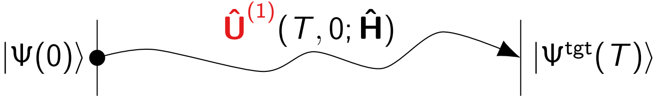


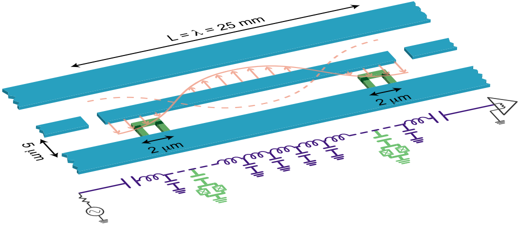
single trajectory can be enough!

$\Rightarrow$ Goerz, Jacobs. arXiv:1801.04382

Charting the circuit QED design landscape using optimal control theory

Goerz et al. npj Quantum Information 3, 37 (2017)

two transmons with shared transmission line bus:



Blais et al. PRA 75, 032329 (2007)

$$\hat{\mathbf{H}} = \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 + \frac{\alpha_1}{2} \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \frac{\alpha_2}{2} \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2$$
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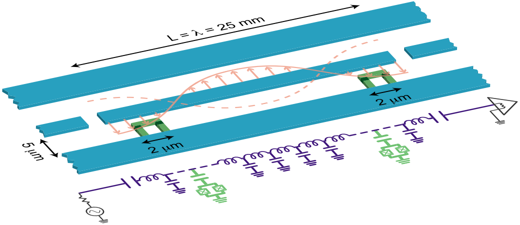

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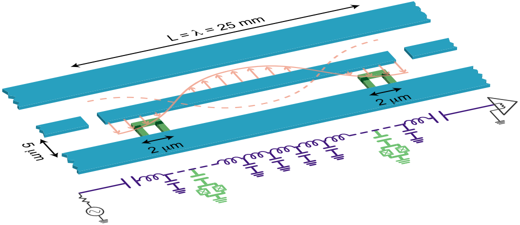

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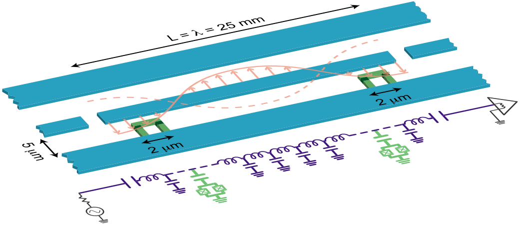

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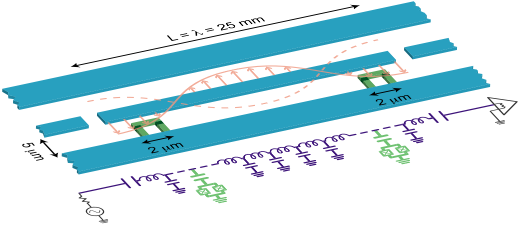

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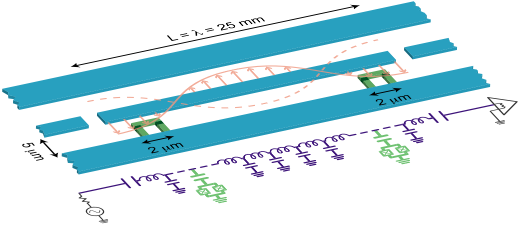

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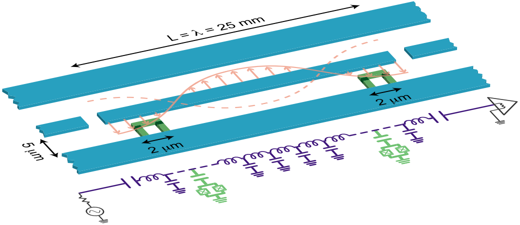

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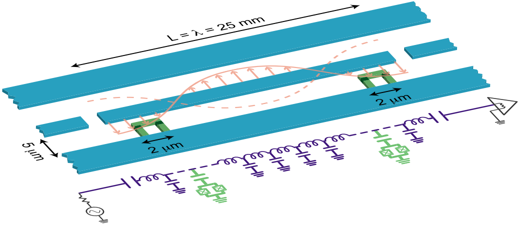

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What are the best parameters for a quantum computer?

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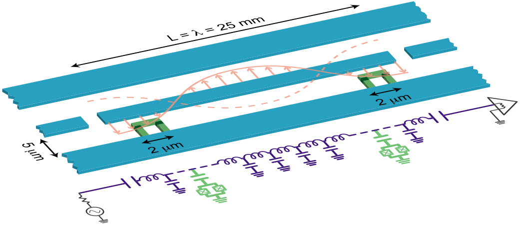

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parameter landscape: Δ_c/g, Δ_2/α

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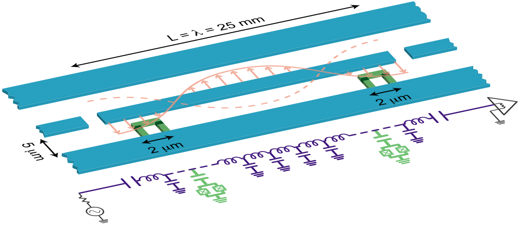

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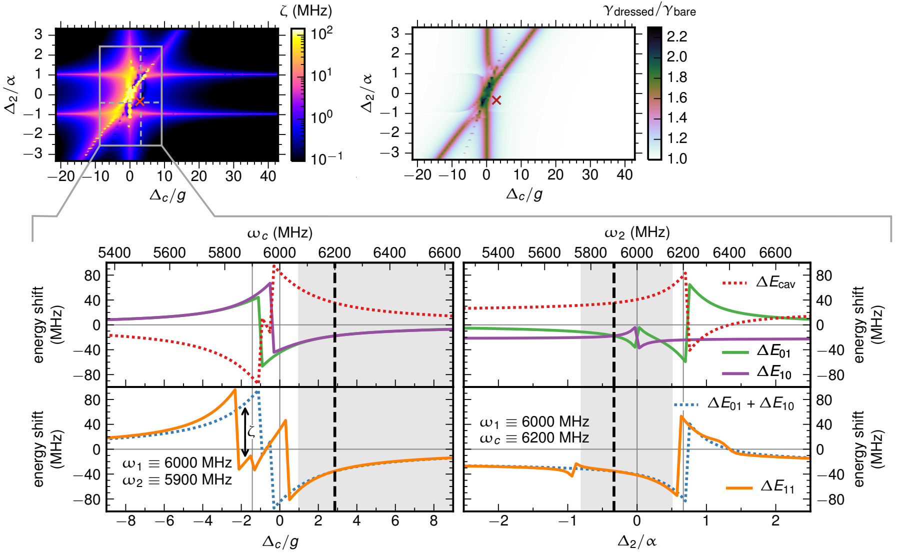
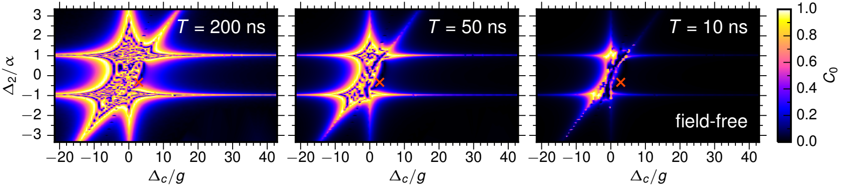
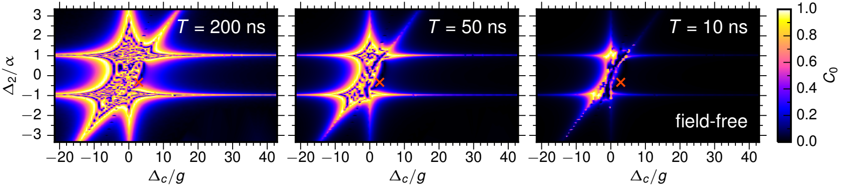
Logical basis: eigenstates of $\hat{\mathbf{H}}$ ("dressed states") properties for field-free Hamiltonian:

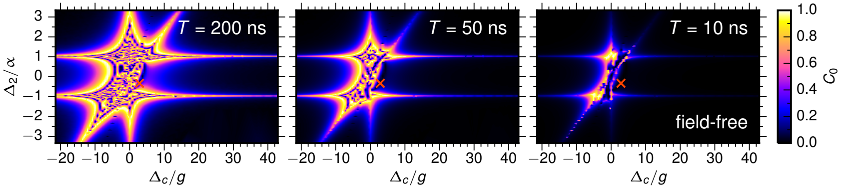




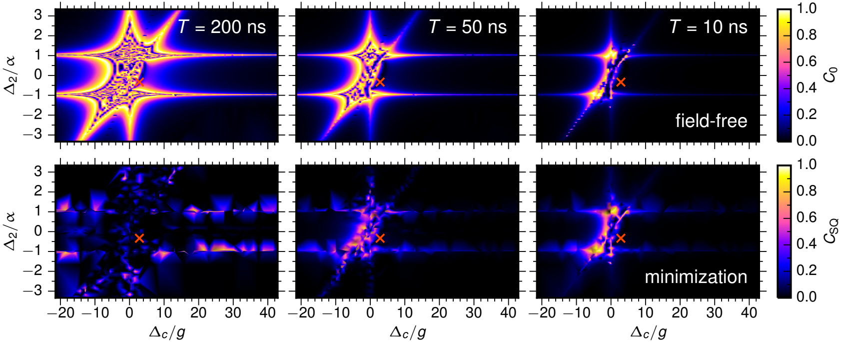
T = 200 ns
T = 50 ns
T = 10 ns
field-free
Δ_2/α
Δ_c/g
C_0

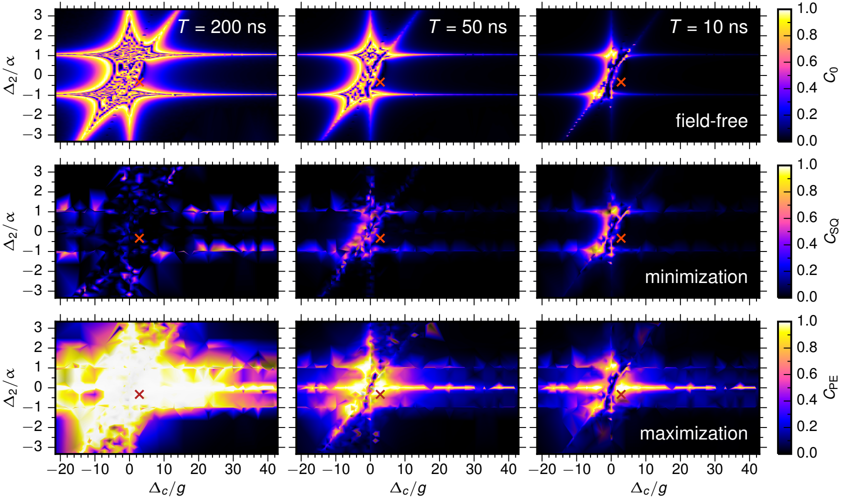


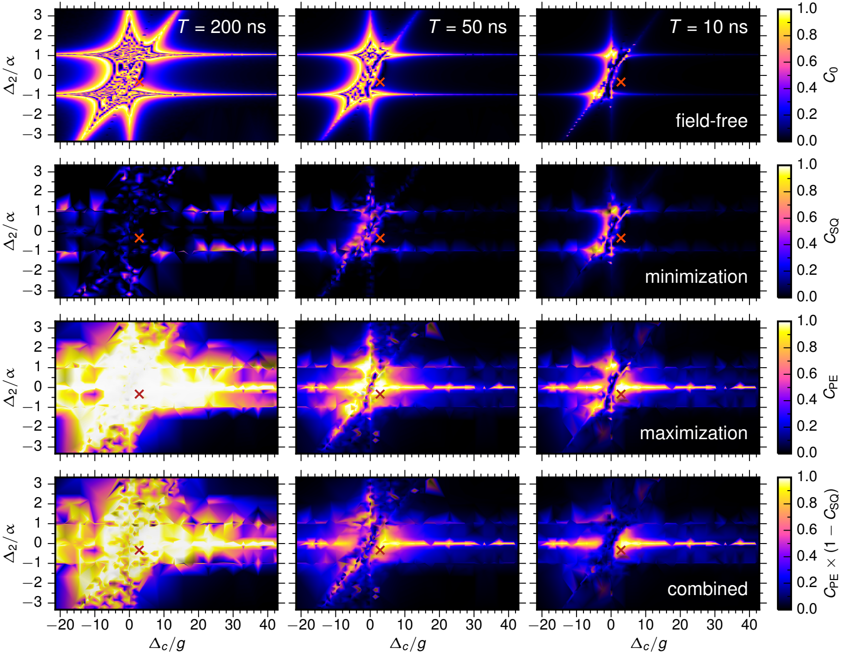
universal quantum computing:
perfect entangler *and* local gates



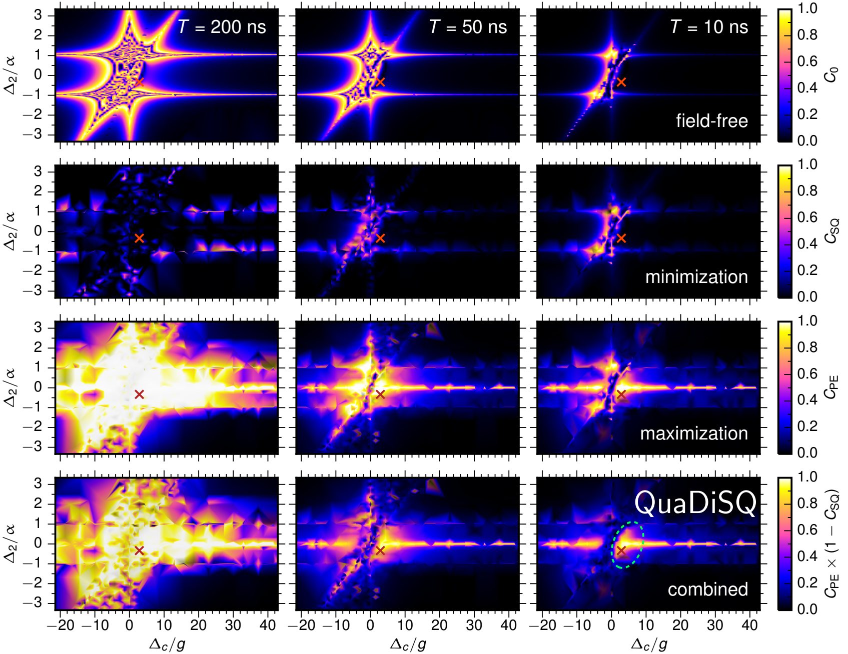
optimize for maximum / minimum entanglement


T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
Δ2/α
Δc/g
C0
CSQ


T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
maximization
Δ_2/α
Δ_c/g
C_0
C_{SQ}
C_{PE}



T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
maximization
combined
Δ_2/α
Δ_c/g
C_0
C_{SQ}
C_{PE}
$C_{PE} \times (1 - C_{SQ})$



T = 200 ns
T = 50 ns
T = 10 ns
field-free
minimization
maximization
QuaDiSQ
combined
Δ2/α
Δc/g
C0
CSQ
CPE
CPE × (1 − CSQ)

optimize for *specific* set of universal gates:

- Hadamard, Phase single qubit
- BGATE entangler

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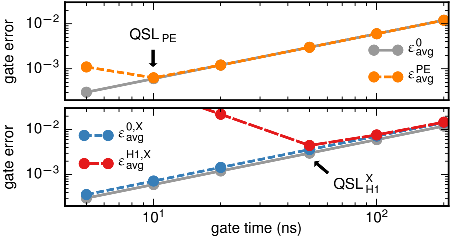

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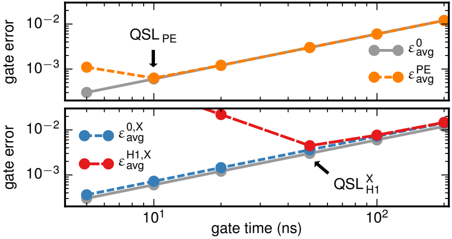
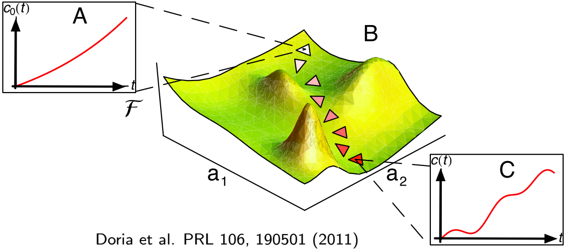


minimum gate duration: 50 ns
gate errors:

- 10^{-4} without dissipation
- 10^{-3} with dissipation

optimal control methods

gradient-free optimization



Doria et al. PRL 106, 190501 (2011)

e.g. Nelder-Mead (simplex), genetic algorithms. . .

gradient-ascent pulse engineering

$$J_T = 1 - \left\langle\!\left\langle \hat{\boldsymbol{\rho}}(0) \middle| \mathcal{E}_1^\dagger \dots \mathcal{E}_j^\dagger \dots \mathcal{E}_N^\dagger \middle| \hat{\mathbf{P}}_{\text{tgt}} \right\rangle\!\right\rangle$$

with $\mathcal{E}_j \equiv \mathcal{E}(t_j, t_{j-1}; u_{ij})$ and $u_{ij} \equiv u_i([t_{j-1}, t_j])$

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update $$\Delta u_{ij} \propto \frac{\partial J_T}{\partial u_{ij}} = -\left\langle\!\!\left\langle \hat{\mathsf{P}}^{(0)}(t_j) \middle| \frac{\partial \mathcal{E}_j}{\partial u_{ij}} \middle| \hat{\boldsymbol{\rho}}^{(0)}(t_{j-1}) \right\rangle\!\!\right\rangle$$

[Khaneja et al. J. Magnet. Res. 172, 296 (2005)]

gradient-ascent pulse engineering

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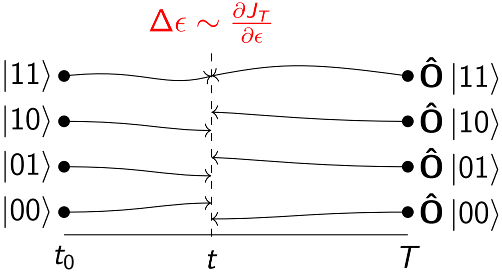

Krotov's method

auxiliary functional

$$J = J_T + \sum_i \frac{\lambda_i}{S_i(t)} \int_0^T \overbrace{\left|u_i^{(1)}(t) - \underbrace{u_i^{(0)}(t)}_{u_i^{\text{ref}}}\right|^2}^{|\Delta u_i(t)|^2} \, \mathrm{d}t$$

different functional in every iteration!

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update $$\Delta u_i(t) = \frac{S_i(t)}{\lambda_i} \mathfrak{Im} \left\langle\!\!\left\langle \hat{\boldsymbol{\Xi}}^{(0)}(t) \middle| \frac{\partial \mathcal{L}}{\partial u_i(t)} \middle| \hat{\boldsymbol{\rho}}^{(1)}(t) \right\rangle\!\!\right\rangle$$

boundary condition $\hat{\boldsymbol{\Xi}}^{(0)}(T) = \frac{J_T}{\langle\!\langle \rho |} = \hat{\mathbf{P}}_{\text{tgt}}$

[Reich et al. J. Chem. Phys. 136, 104103 (2012)]

“hybrid” (multi-stage) methods

1. Start with analytical formula, optimize free parameter with simplex
2. Use simplex-optimized control as starting point for gradient-based method

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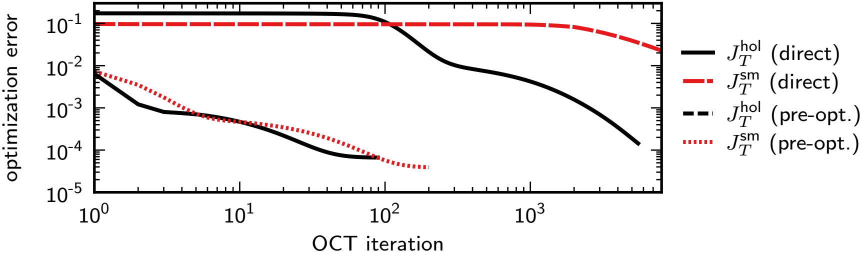
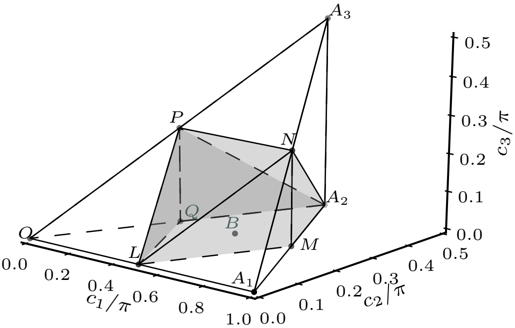


⇒ Goerz et al. EPJ Quantum Tech. 2, 21 (2015)

advanced functionals

optimizing for entanglement

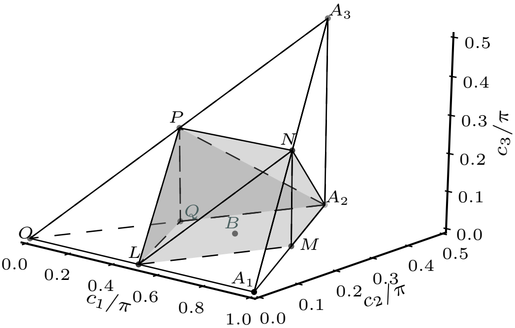
optimizing for entanglement



Cartan decomposition:

$$\hat{\mathbf{U}} = \hat{\mathbf{k}}_1 \exp\left[\frac{\mathrm{i}}{2}\left(c_1\hat{\boldsymbol{\sigma}}_x\hat{\boldsymbol{\sigma}}_x + c_2\hat{\boldsymbol{\sigma}}_y\hat{\boldsymbol{\sigma}}_y + c_3\hat{\boldsymbol{\sigma}}_z\hat{\boldsymbol{\sigma}}_z\right)\right]\hat{\mathbf{k}}_2$$

optimizing for entanglement



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$\Rightarrow$ application: Goerz et al. Phys. Rev. A 91, 062307 (2015)

quantum gates in open quantum systems

Hilbert space

$$J_T = 1 - \frac{1}{16} \left| \sum_{i=1}^{4} \left\langle i \,\middle|\, \hat{\mathbf{O}}^{\dagger} \hat{\mathbf{U}} \,\middle|\, i \right\rangle \right|^2 ; \quad |i\rangle \in \{ |00\rangle, |01\rangle, |10\rangle, |11\rangle \}$$

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Liouville space

$\Rightarrow$ Goerz et al. NJP 16, 055012 (2014)

$$J_T = 1 - \sum_{i=1}^{3}\frac{w_i}{\mathrm{tr}[\hat{\boldsymbol{\rho}}_i^2]}\mathfrak{Re}\left\{\mathrm{tr}\left[\hat{\boldsymbol{\rho}}_i^{\mathrm{tgt}}\hat{\boldsymbol{\rho}}_i(T)\right]\right\}$$

$$\hat{\rho}_1 = \frac{1}{20}\begin{pmatrix}8 & 0 & 0 & 0\\ 0 & 6 & 0 & 0\\ 0 & 0 & 4 & 0\\ 0 & 0 & 0 & 2\end{pmatrix},\ \hat{\rho}_2 = \frac{1}{4}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\end{pmatrix},\ \hat{\rho}_3 = \frac{1}{4}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

populations phases subspace

Summary

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  - fast protocols
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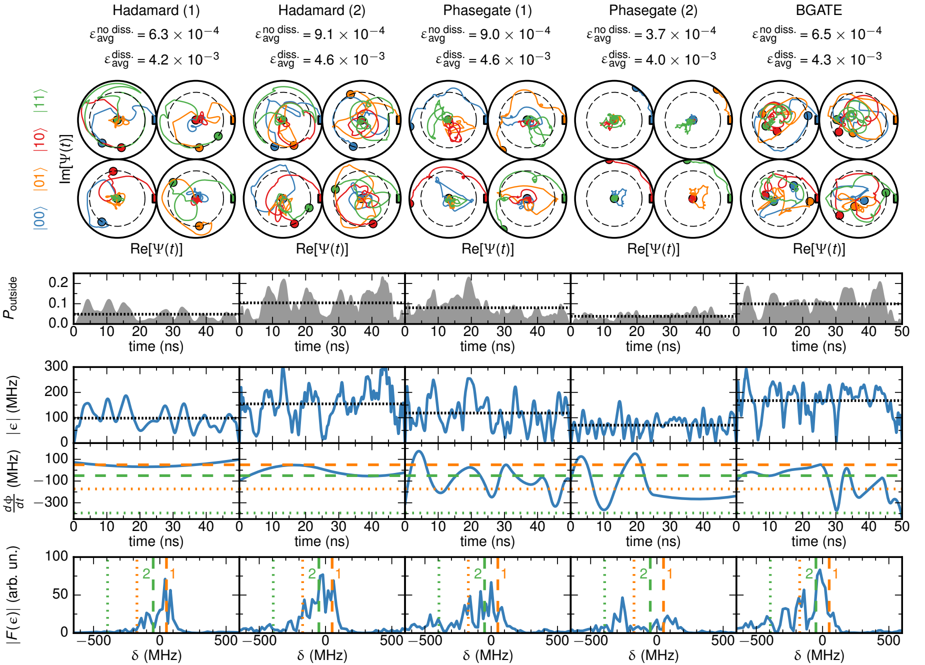
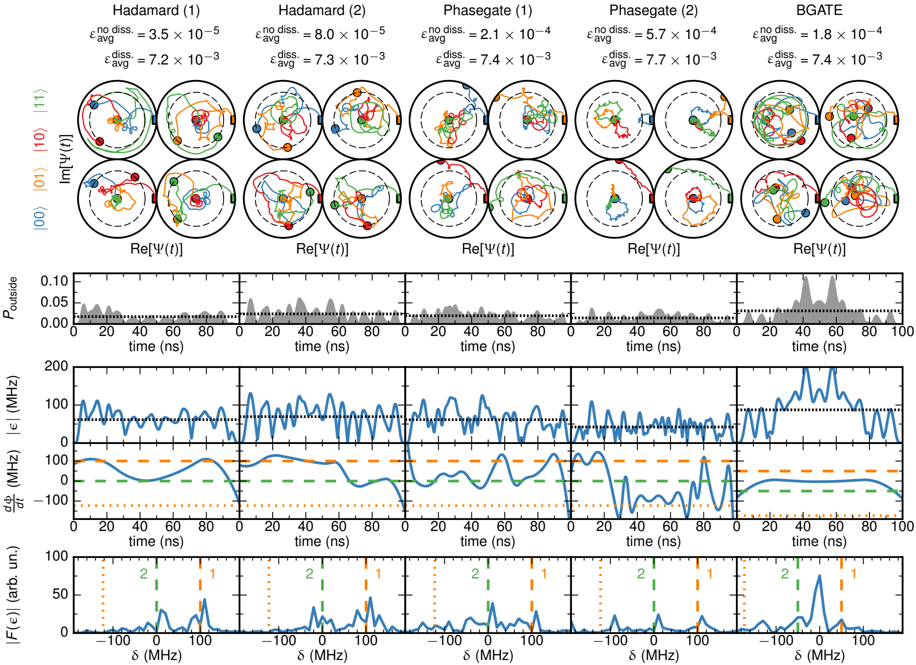
Thank You


Hadamard (1)
$\varepsilon_{avg}^{no\ diss.} = 6.3 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.2 \times 10^{-3}$
Hadamard (2)
$\varepsilon_{avg}^{no\ diss.} = 9.1 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.6 \times 10^{-3}$
Phasegate (1)
$\varepsilon_{avg}^{no\ diss.} = 9.0 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.6 \times 10^{-3}$
Phasegate (2)
$\varepsilon_{avg}^{no\ diss.} = 3.7 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.0 \times 10^{-3}$
BGATE
$\varepsilon_{avg}^{no\ diss.} = 6.5 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 4.3 \times 10^{-3}$
$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$
$\mathrm{Im}[\Psi(t)]$
$\mathrm{Re}[\Psi(t)]$
$P_{outside}$
time (ns)
$|\epsilon|$ (MHz)
$\frac{d\phi}{dt}$ (MHz)
$|F(\epsilon)|$ (arb. un.)
δ (MHz)


Hadamard (1)
$\varepsilon_{avg}^{no\ diss.} = 3.5 \times 10^{-5}$
$\varepsilon_{avg}^{diss.} = 7.2 \times 10^{-3}$
Hadamard (2)
$\varepsilon_{avg}^{no\ diss.} = 8.0 \times 10^{-5}$
$\varepsilon_{avg}^{diss.} = 7.3 \times 10^{-3}$
Phasegate (1)
$\varepsilon_{avg}^{no\ diss.} = 2.1 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 7.4 \times 10^{-3}$
Phasegate (2)
$\varepsilon_{avg}^{no\ diss.} = 5.7 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 7.7 \times 10^{-3}$
BGATE
$\varepsilon_{avg}^{no\ diss.} = 1.8 \times 10^{-4}$
$\varepsilon_{avg}^{diss.} = 7.4 \times 10^{-3}$
$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$
$\mathrm{Im}[\Psi(t)]$
$\mathrm{Re}[\Psi(t)]$
$P_{outside}$
time (ns)
$|\epsilon|$ (MHz)
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$|F(\epsilon)|$ (arb. un.)
δ (MHz)
2
1

obtained perfect entanglers in the Weyl chamber: