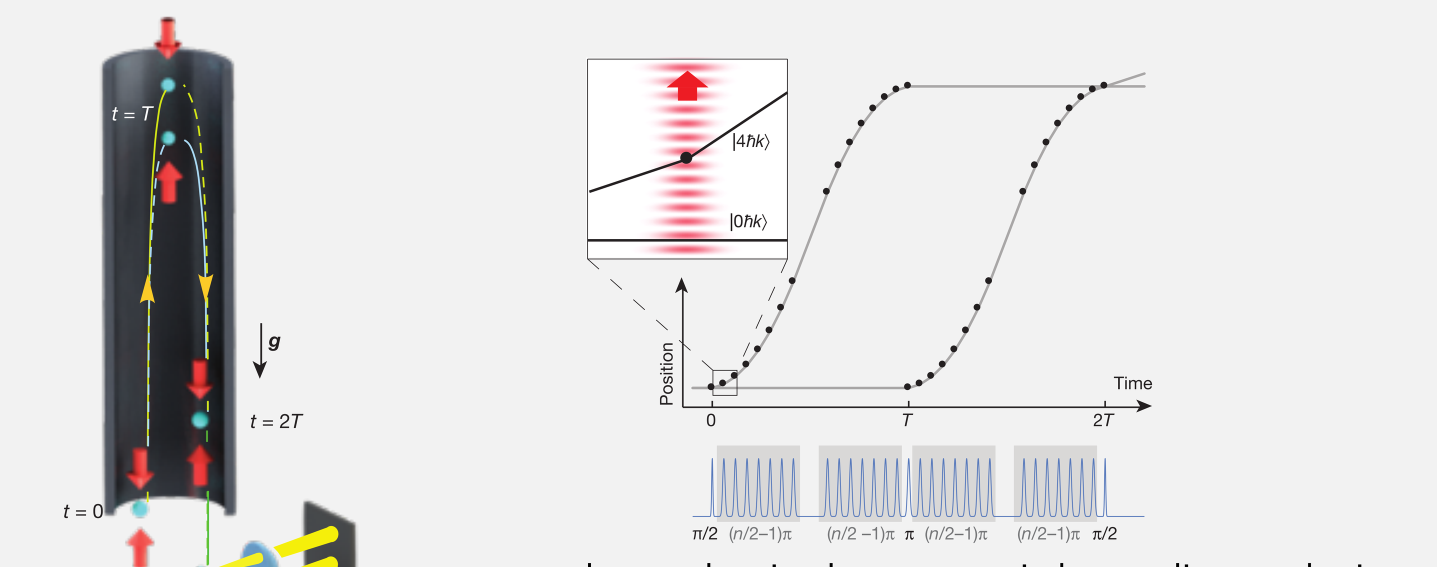
Optimal Control for Robust Atomic Fountain Interferometry

Michael H. Goerz[1], Paul D. Kunz[1], Mark A. Kasevich[2], Vladimir S. Malinovsky[1]

[1]*U.S. Army Research Lab, Computational and Information Science Directorate, Adelphi, MD* [2]*Department of Physics, Stanford University, Stanford, CA*

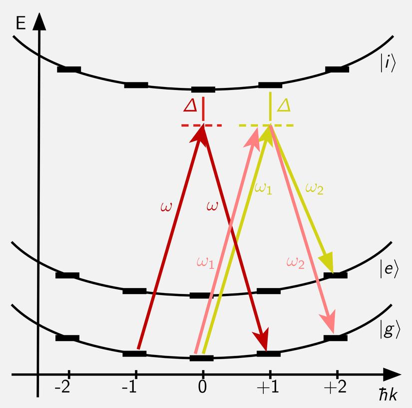
① Atomic Fountain Interferometer



laser pulses implement atomic beamsplitter and mirror

Applications: gravitational sensing, inertial navigation, test of equivalence principle

Momentum space Hamiltonian for linear-chirp Bragg pulses [2]

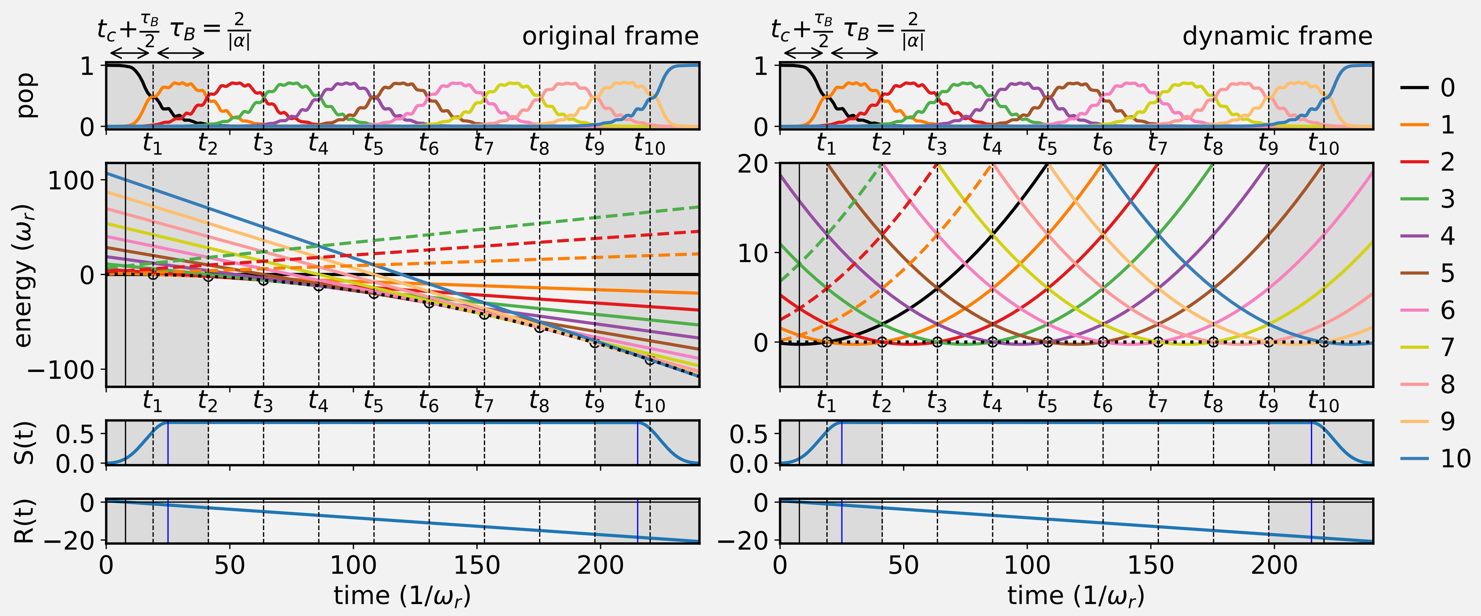


- adiabatic elimination of $|i\rangle$; pulse with linear chirp rate α:

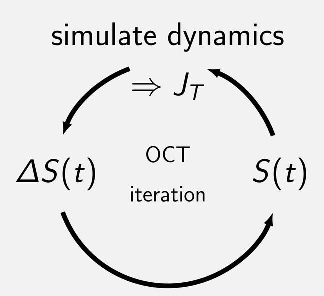
$$\hat{\mathsf{H}} = \sum_n \omega_r \big(n^2 + \underbrace{\alpha(t-t_c)}_{R(t)} n - \delta_0 n\big) |n\rangle\langle n| - \mu S(t) \sum_n \big(|n\rangle\langle n+1| + \text{H.c.}\big); \quad \delta_0 = 0, \mu = 1$$

- dynamic frame transformation: $\hat{\mathsf{U}}(t) = e^{i\phi(t)}$;

$$\phi(t) = -\frac{\alpha^2}{12}t^3 + \frac{t_c\alpha^2}{4}t^2 - \frac{t_c^2\alpha^2+1}{4}$$



② Optimal Control Methods



numerical optimal control: iteratively minimize functional J_T:

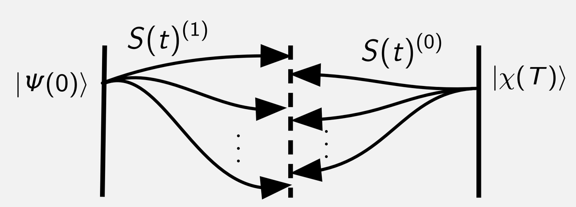
$$J_{T,\text{sm}} = \varepsilon_{\text{sm}} = 1 - \left|\langle \Psi^{\text{tgt}} \,|\, \Psi(T)\rangle\right|^2 \quad \text{phase-insensitive}$$
$$J_{T,\text{re}} = \varepsilon_{\text{re}} = 1 - \text{Re}\,\langle \Psi^{\text{tgt}} \,|\, \Psi(T)\rangle \quad \text{phase-sensitive}$$

Gradient-based optimization: Krotov's method [3]

update: $\Delta S(t) \propto \text{Im}\left\langle \chi^{(0)}(t) \left| \frac{\partial H}{\partial S(t)} \right| \Psi^{(1)}(t) \right\rangle$; $\quad \left|\chi^{(0)}(T)\right\rangle = \frac{\partial J_T}{\partial \langle \Psi^{\text{tgt}}|}$

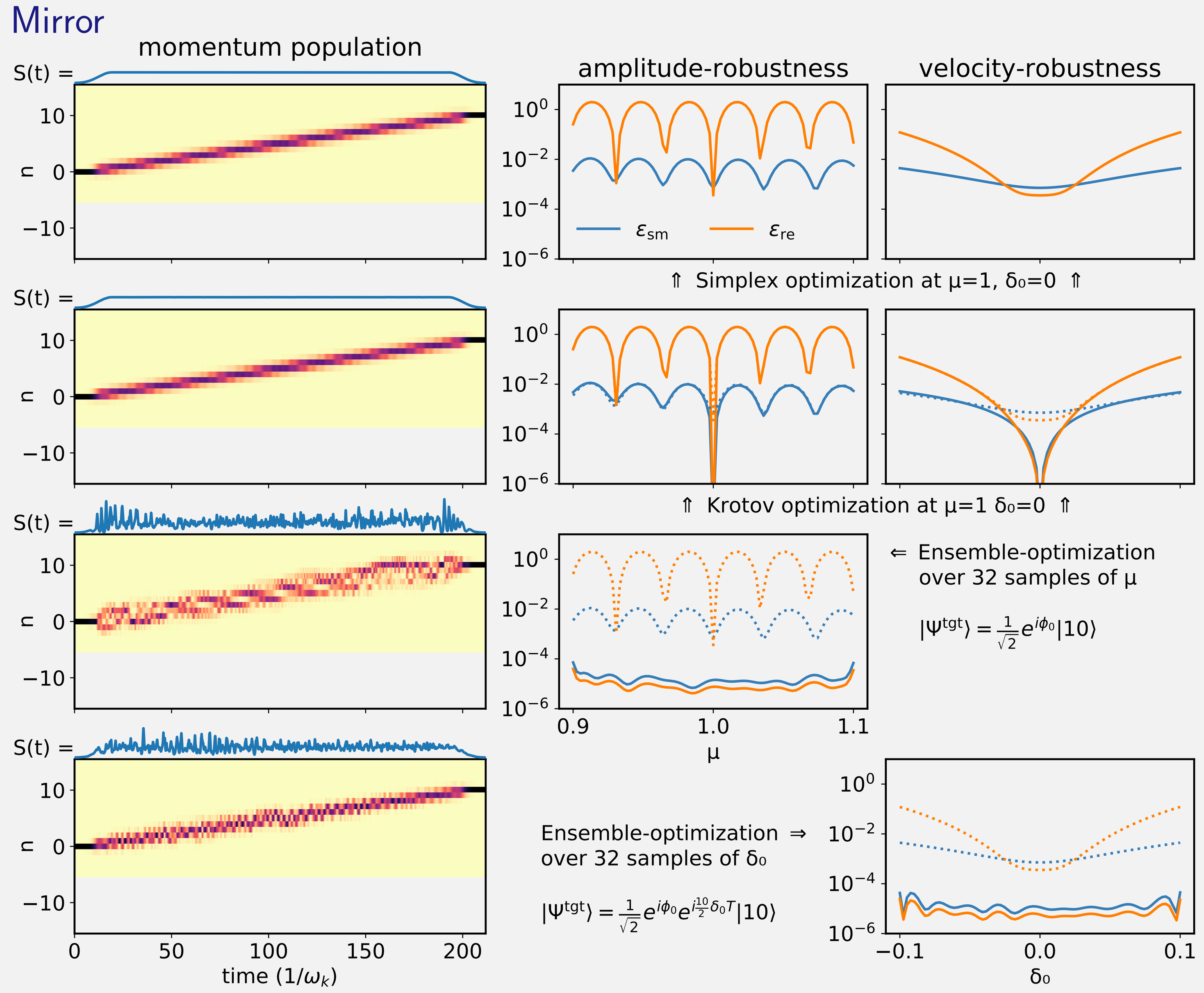
Ensemble optimization [4]

Optimize over average of ensemble of variations of the Hamiltonian (μ, δ_0)

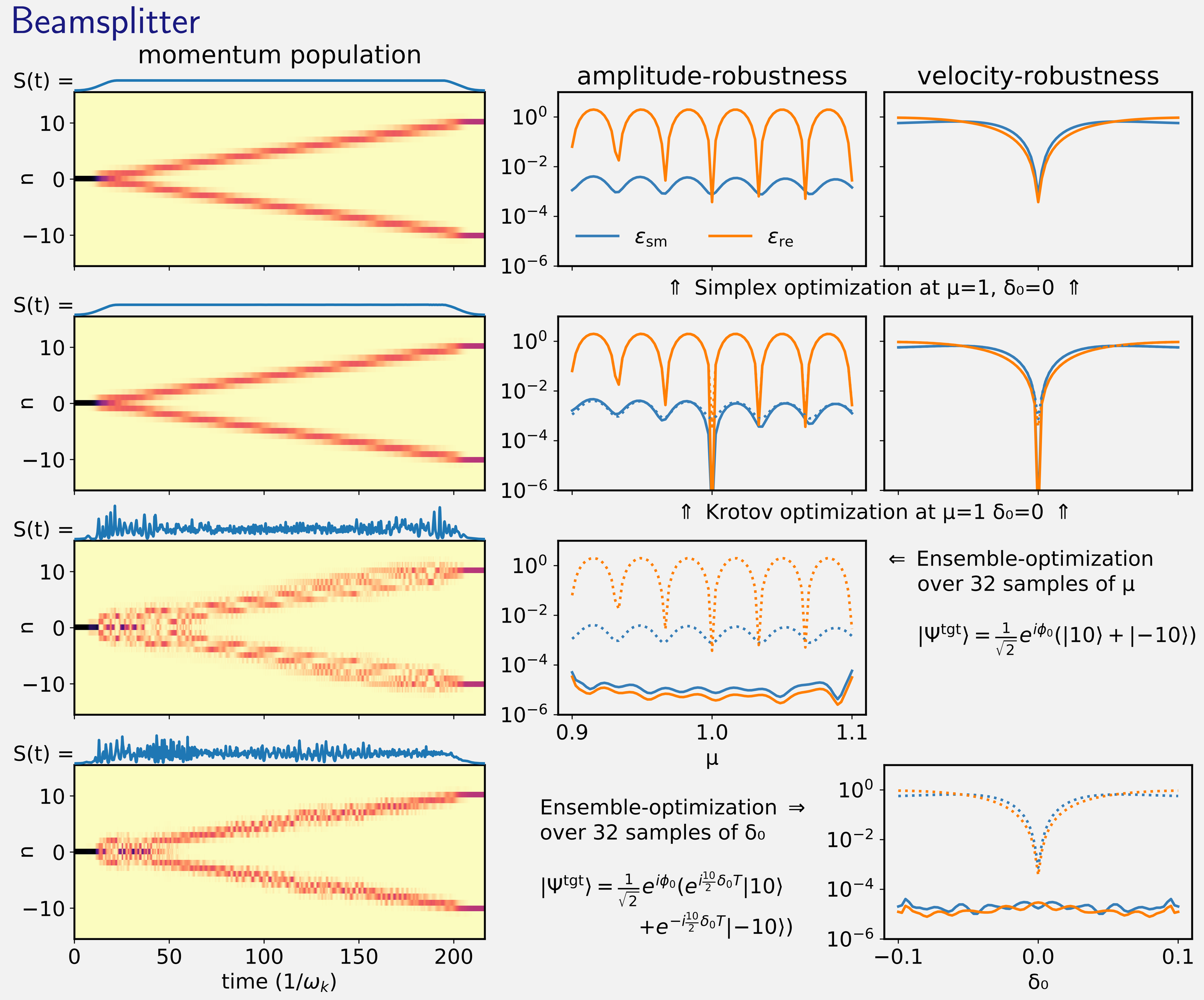


③ Near-adiabatic scheme, $\alpha = -0.1$

Mirror

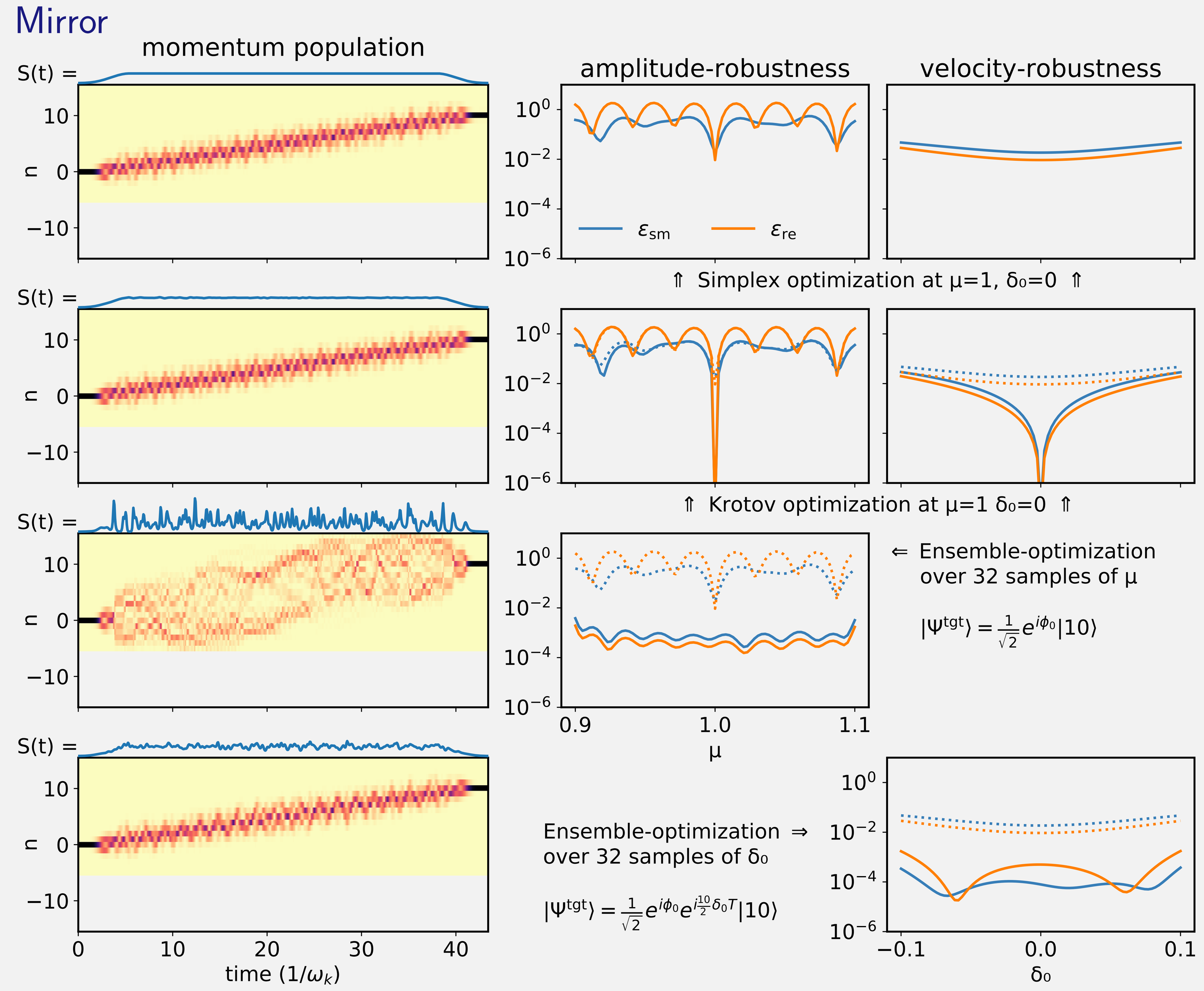


Beamsplitter

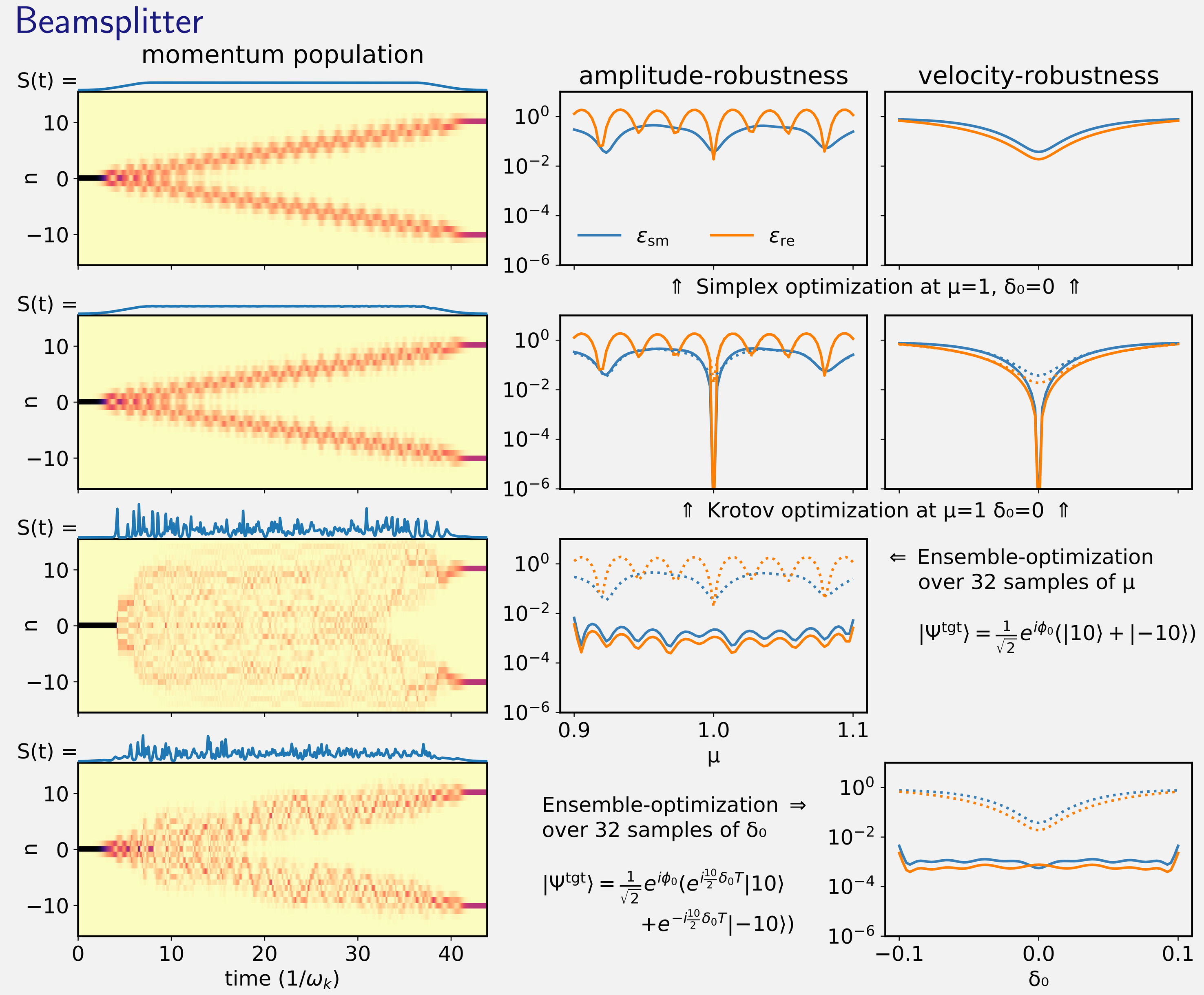


④ Compressed scheme, $\alpha = -0.5$

Mirror



Beamsplitter



References

[1] Kovachi et al. *Nature* **528**, 530 (2015)
[2] Malinovky, Berman, *Phys. Rev. A* **68**, 023610 (2003)
[3] Reich et al. *J. Chem. Phys.* **136**, 104103 (2012)
[4] Goerz et al. *Phys. Rev. A* **90**, 032329 (2014)

Outlook

- Robust pulses for *both* sources of errors (μ, δ_0) simultaneously?
- Pulse modularity: arbitrarily high momentum transfer (dynamic frame transformation)