

Optimal Control for Entangling Quantum Gates

Michael Goerz

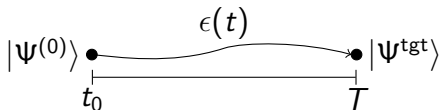
Stanford/Army Research Lab

UMBC Physics Colloquium
April 19, 2017

the quantum optimal control problem

quantum technology

steer quantum system in some desired way

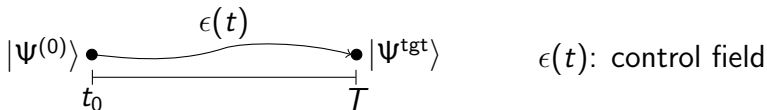


$\epsilon(t)$: control field

the quantum optimal control problem

quantum technology

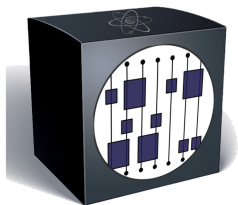
steer quantum system in some desired way



examples:

- **photo-chemistry:** form atomic bonds
- **medical imaging:** orient nuclear spin for max resolution
- **quantum networks:** prepare non-classical states
- **quantum computing:** apply logical operation (“gate”)
- ...

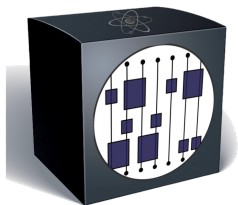
optimizing quantum gates



$$|\psi\rangle = \alpha_0 \underbrace{|0 \dots 1\rangle}_{N \text{ qubits}} + \dots + \alpha_{2^N} |1 \dots 1\rangle$$

reduce to two-qubit gates: 4×4 matrix

optimizing quantum gates



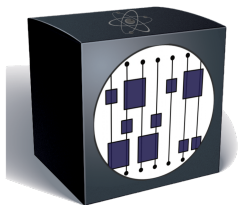
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$$|\Psi\rangle \rightarrow \hat{\mathbf{O}} |\Psi\rangle, \text{ e.g.}$$

$$\hat{\mathbf{O}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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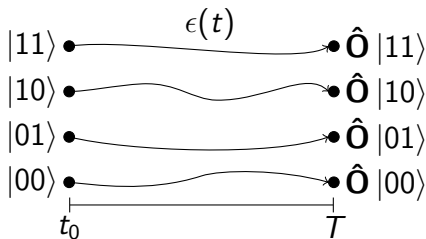


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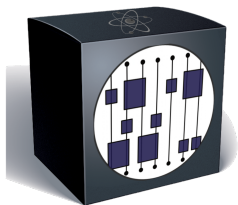
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simultaneous targets (basis states)!

optimizing quantum gates



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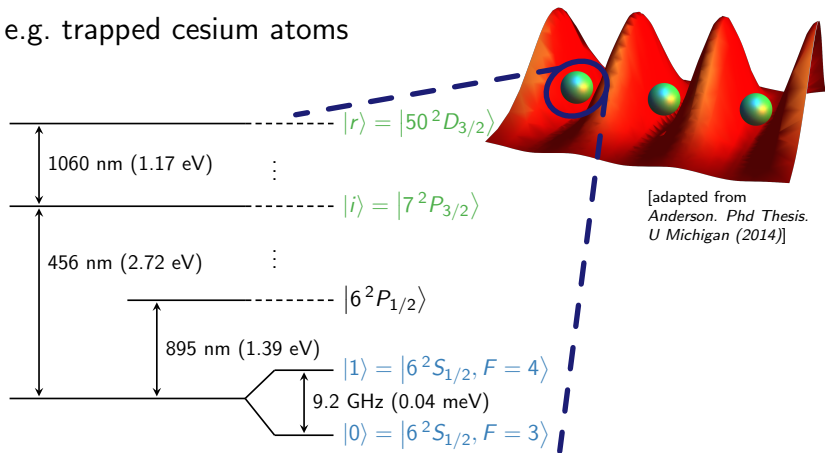
reduce to two-qubit gates: 4×4 matrix

Implementations:

- trapped atoms
- superconducting circuits
- NV centers
- quantum dots
- ...

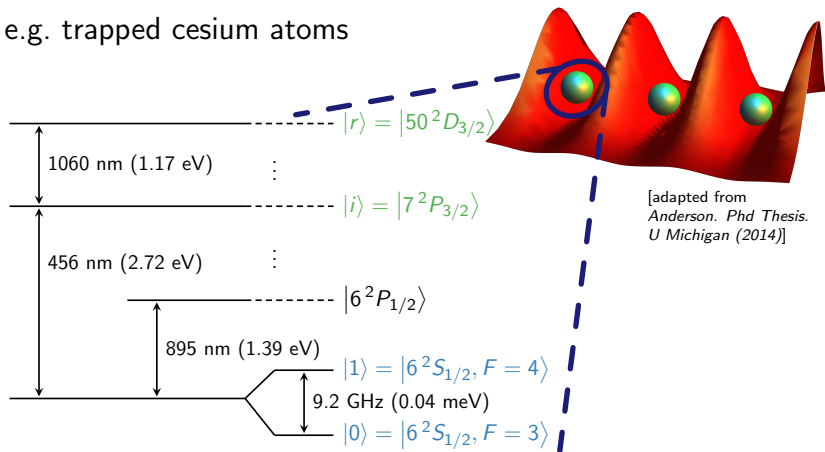
logical subspace

e.g. trapped cesium atoms



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logical subspace embedded in larger total Hilbert space!

analytical:

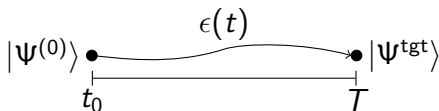
- geometric control – low dimension
- adiabatic schemes (e.g. STIRAP) – slow
- open quantum systems? noise? fundamental limits?

numerical optimal control

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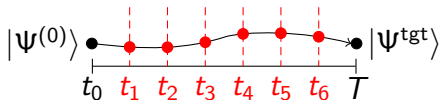


numerical optimal control

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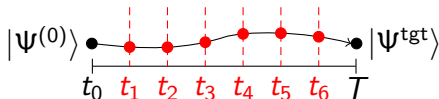


numerical optimal control

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numerical:



minimize functional J_T

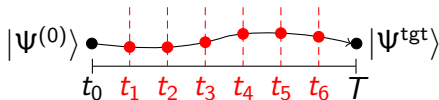
e.g. $J_T = 1 - \frac{1}{d^2} \sum_{k=1}^d |\langle \psi_k^{\text{tgt}} | \psi_k(T) \rangle|^2$

numerical optimal control

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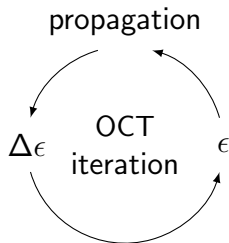
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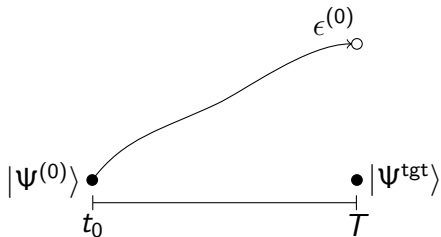
$$\text{e.g. } J_T = 1 - \frac{1}{d^2} \sum_{k=1}^d |\langle \Psi_k^{\text{tgt}} | \Psi_k(T) \rangle|^2$$

\Rightarrow iterative scheme



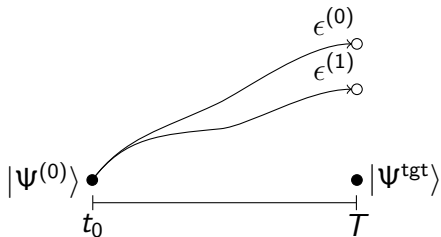
Optimization Methods

gradient-free optimization



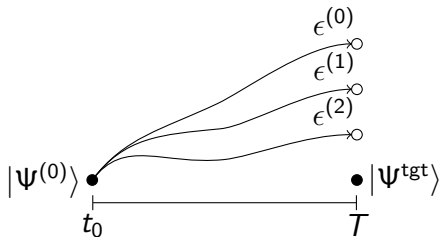
only evaluate fig. of merit J_T

gradient-free optimization



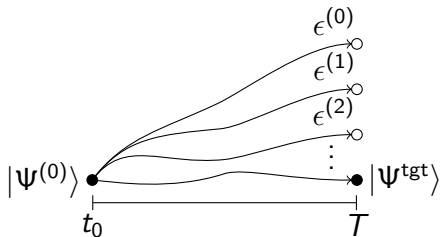
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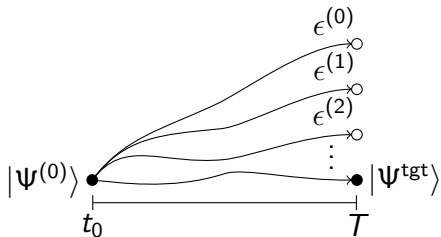
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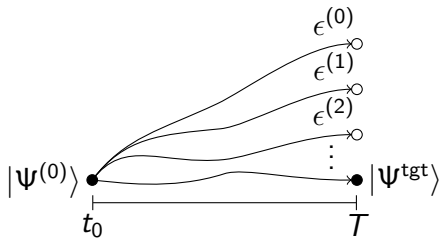
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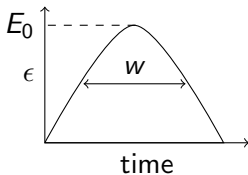
■ any J_T

gradient-free optimization

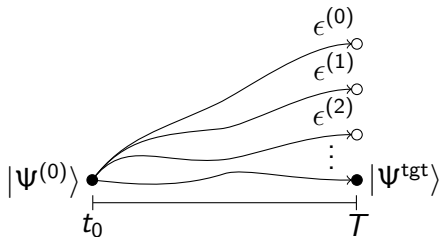


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- any J_T
- good for small number of control parameters



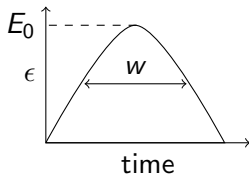
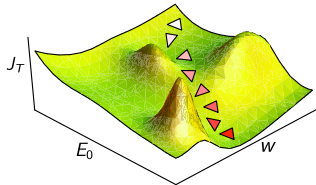
gradient-free optimization



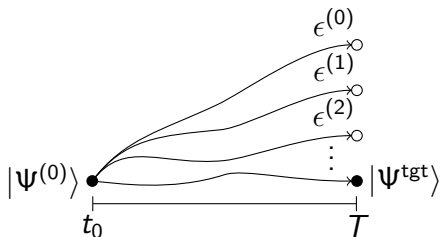
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Nelder-Mead simplex:



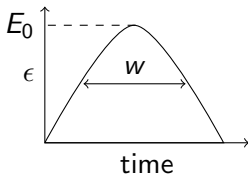
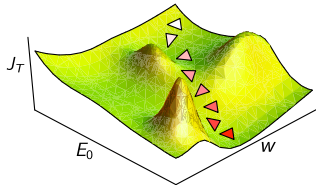
gradient-free optimization



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Nelder-Mead simplex:



easy to use: `scipy.optimize`, Matlab, ...

GRAPE/LBFGS

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- $J_T \sim \langle \Psi^{\text{tgt}} | \Psi(T) \rangle = \langle \Psi^{\text{tgt}} | \hat{\mathbf{U}}_{nt} \dots \hat{\mathbf{U}}_1 | \Psi_0 \rangle$

GRAPE/LBFGS

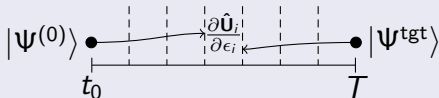
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update scheme

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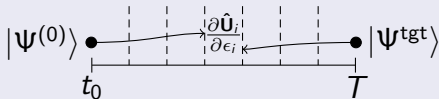


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Khaneja et al. J. Magnet. Res. 172, 296 (2005)

library implementation: L-BFGS-B

Krotov's method

- variational calculus, for *continuous* $\epsilon(t)$

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 $(|\Psi(t)\rangle = \hat{\mathbf{U}}(t, 0; \epsilon(t)) |\Psi_0\rangle)$

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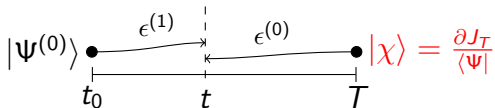
The diagram illustrates the update scheme for the field $\epsilon(t)$. It shows a horizontal timeline from t_0 to T . At t_0 , the state is $|\Psi^{(0)}\rangle$. At T , the state is $|\chi\rangle = \frac{\partial J_T}{\partial \Psi}$. A solid arrow labeled $\epsilon^{(0)}$ points from T back to t . A dashed vertical line is at t . A solid arrow labeled $\epsilon^{(1)}$ points from t_0 to t .

Reich et al. J. Chem. Phys. 136, 104103 (2012)

Krotov's method vs GRAPE

Krotov's method

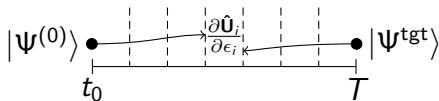
$$\Delta\epsilon(t) \sim \langle \chi^{\text{bw}} | \frac{\partial \hat{H}}{\partial \epsilon} | \psi^{\text{fw}} \rangle$$



- sequential update
- continuous \rightarrow discrete
- guaranteed monotonic convergence
- J_T only in boundary condition

GRAPE

$$\Delta\epsilon_i \sim \frac{\partial J_T}{\partial \epsilon_i} \sim \langle \psi^{\text{bw}} | \frac{\partial \hat{U}_i}{\partial \epsilon_i} | \psi^{\text{fw}} \rangle$$



- concurrent update
- inherently discrete
- parametrization through chain rule

Applications

the quantum speed limit

- progressively decrease gate duration

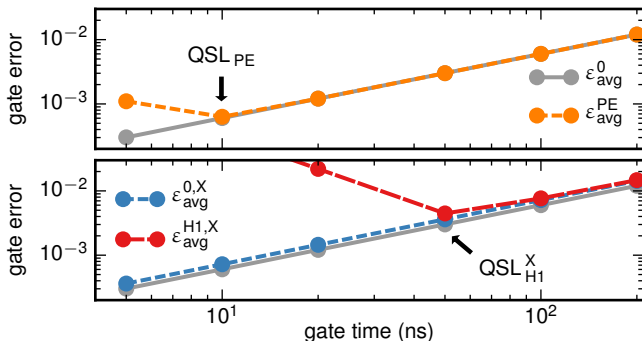
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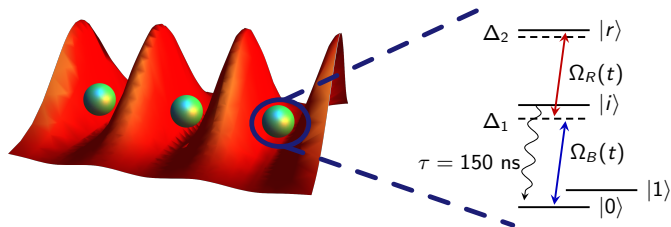
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example: optimization of entangling and local gates
in superconducting transmon qubits

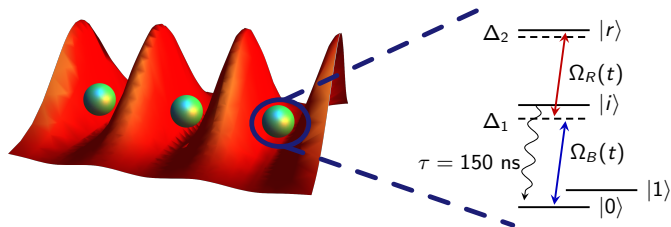


Goerz et al. arXiv:1606.08825 (2017)

robustness to classical fluctuations

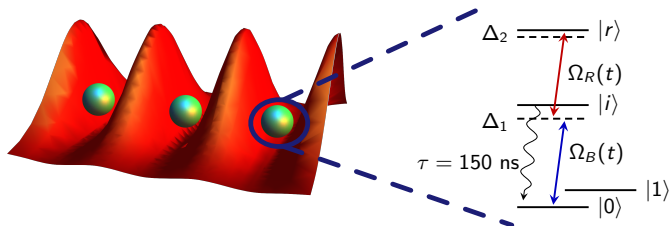


robustness to classical fluctuations



noise sources: fluctuation of Rydberg level, field amplitude

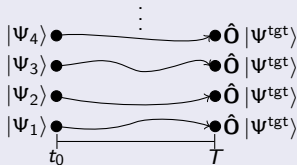
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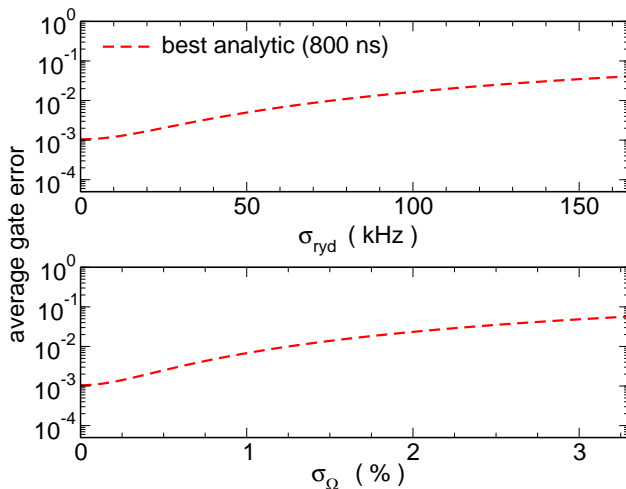
noise sources: fluctuation of Rydberg level, field amplitude

ensemble optimization

simultaneously optimize over multiple copies of the system with different noise realizations

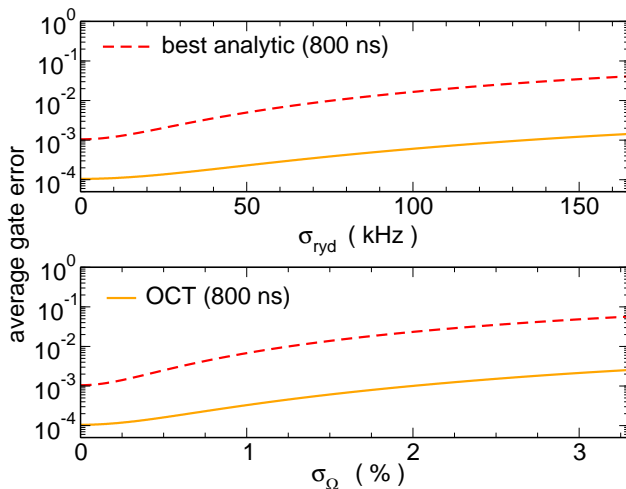


robustness Rydberg gates



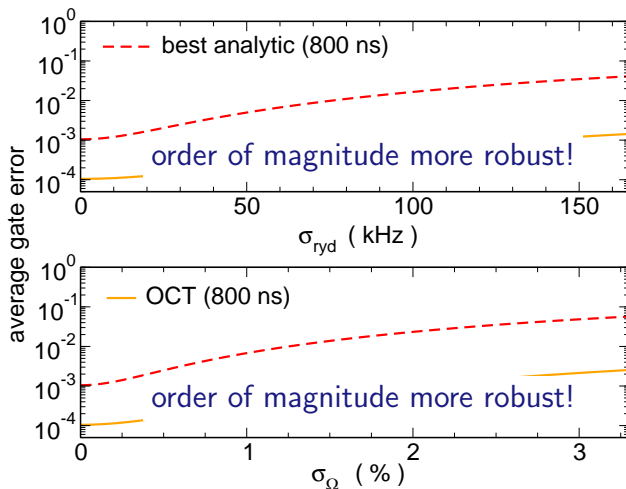
⇒ Goerz, Halperin, Aytac, Koch, Whaley. PRA 90, 032329 (2014)

robustness Rydberg gates



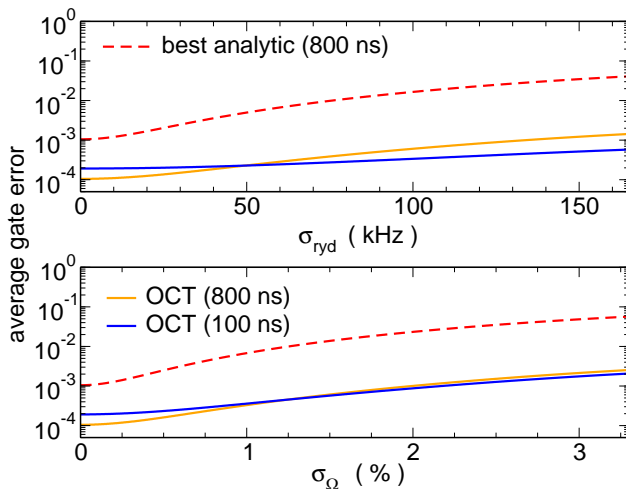
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robustness to dissipation

just optimize density matrices!

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$$J_T = 1 - \sum_{i=1}^3 \frac{w_i}{\text{tr}[\hat{\rho}_i^2]} \Re \left\{ \text{tr} \left[\hat{\rho}_i^{\text{tgt}} \hat{\rho}_{i,n}(T) \right] \right\}$$

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$$\hat{\rho}_1 = \frac{1}{20} \begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \hat{\rho}_2 = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad \hat{\rho}_3 = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

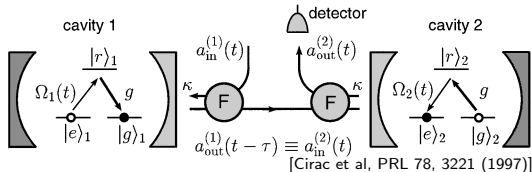
populations phases subspace

⇒ Goerz, Reich, Koch. NJP 16, 055012 (2014).

always 3 states, independent of dimension!

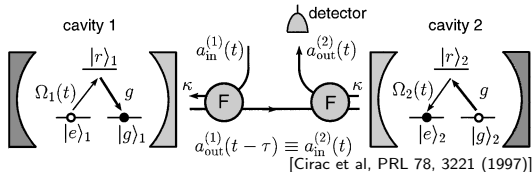
Alternative: MCWF trajectories

trajectory optimization



$$\hat{H} = \hat{H}_1 + \hat{H}_2 + i\kappa(\hat{\mathbf{a}}_1^\dagger \hat{\mathbf{a}}_2 - \hat{\mathbf{a}}_1 \hat{\mathbf{a}}_2^\dagger), \quad \hat{L} = \sqrt{2\kappa}(\hat{\mathbf{a}}_1 + \hat{\mathbf{a}}_2)$$

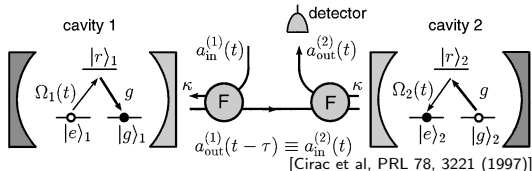
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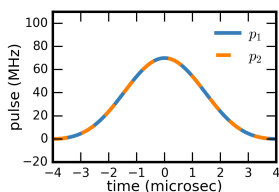
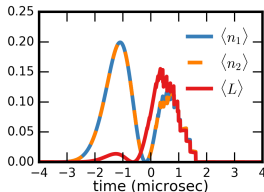
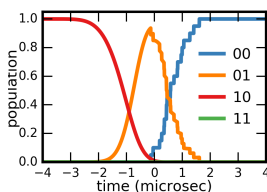
propagate with $\hat{H}_{\text{eff}} = \hat{H} - \frac{i\hbar}{2} \hat{L}^\dagger \hat{L}$,
 jump randomly with probability of $\|\Psi\|$

trajectory optimization

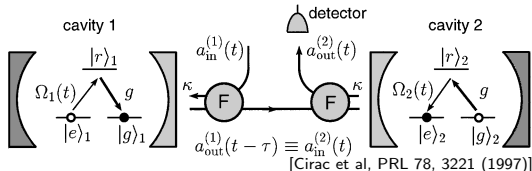


$$\hat{H} = \hat{H}_1 + \hat{H}_2 + i\kappa(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_1 \hat{a}_2^\dagger), \quad \hat{L} = \sqrt{2\kappa}(\hat{a}_1 + \hat{a}_2)$$

propagate with $\hat{H}_{\text{eff}} = \hat{H} - \frac{i\hbar}{2}\hat{L}^\dagger\hat{L}$,
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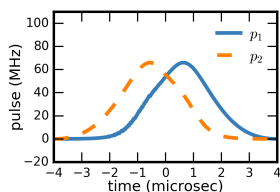
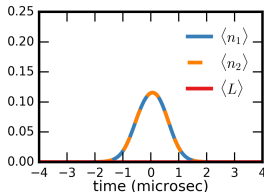
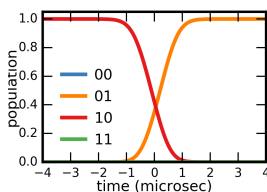


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optimization in the Weyl chamber

- for two-qubit gates: many quantum gates are useful

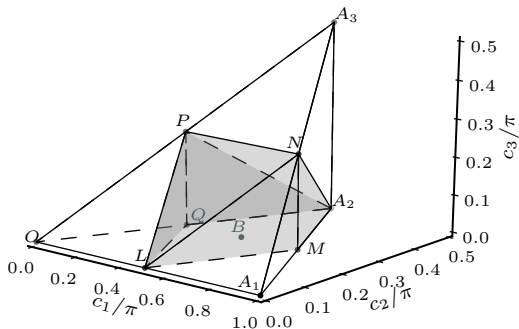
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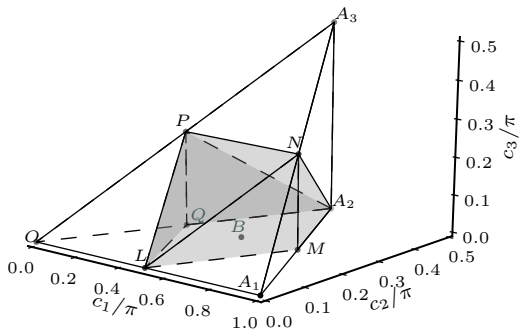
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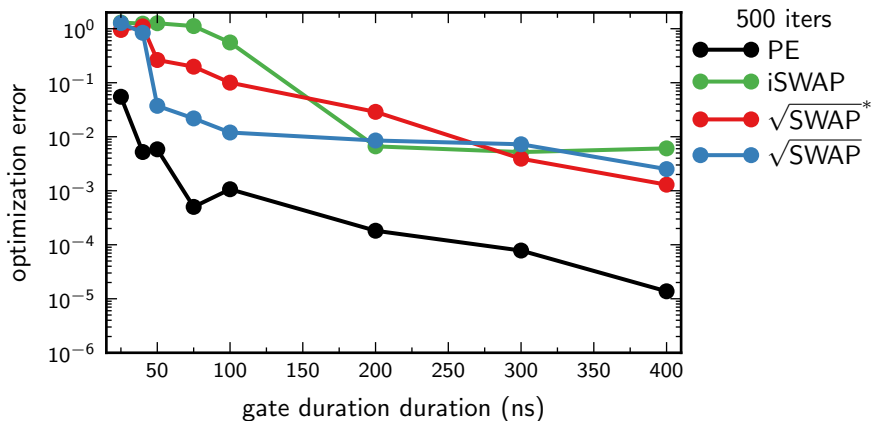
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- optimize for arbitrary perfect entangler

optimization for a perfect entangler

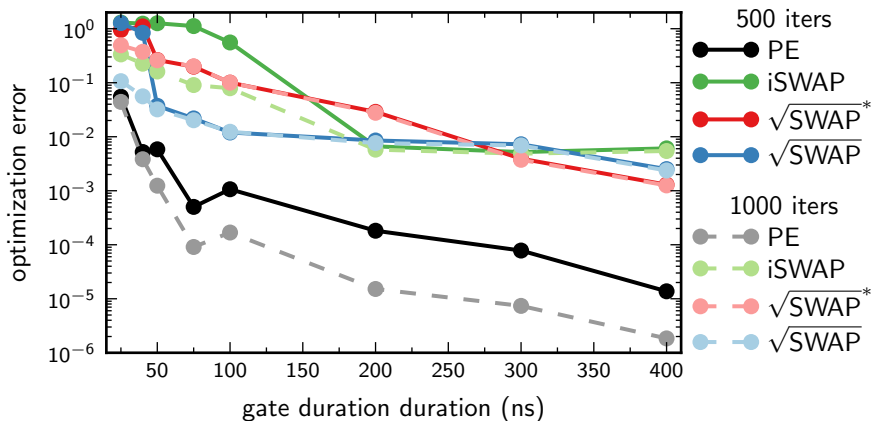
PE optimization for superconducting transmon qubits



⇒ Goerz et al. Phys. Rev. A 91, 062307 (2015)

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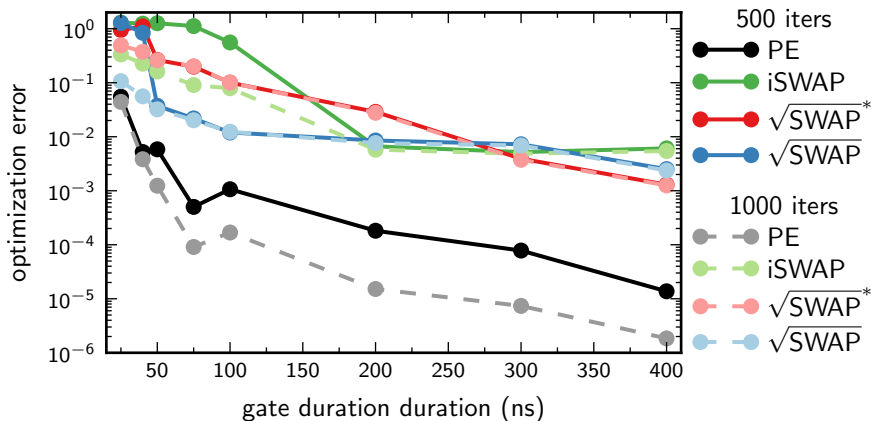
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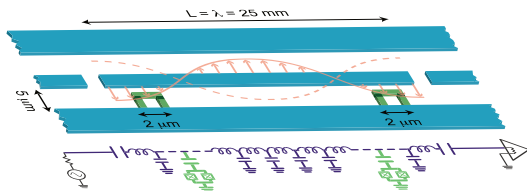


much lower errors and **much better convergence** \Rightarrow faster gates

\Rightarrow Goerz et al. Phys. Rev. A 91, 062307 (2015)

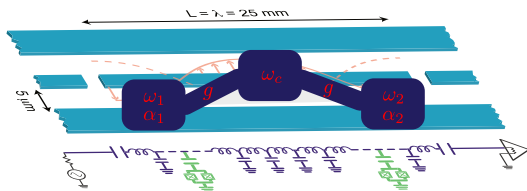
design landscape exploration

two transmon qubits with shared transmission line



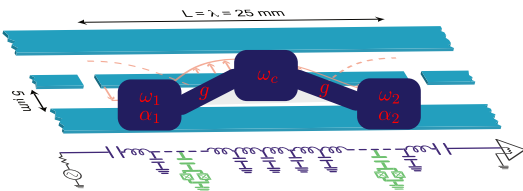
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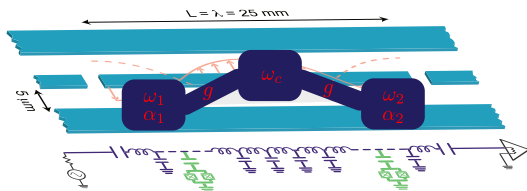
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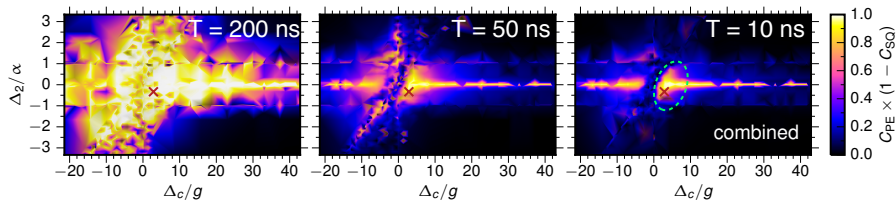
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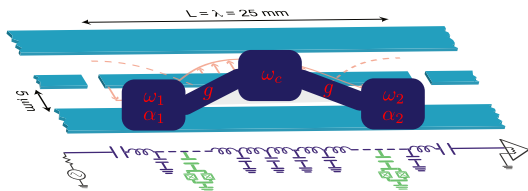


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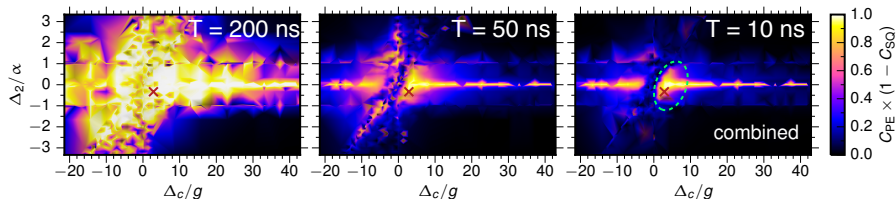


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Identification of new QuaDiSQ regime

⇒ Goerz et al. arXiv:1606.08825 (2016).

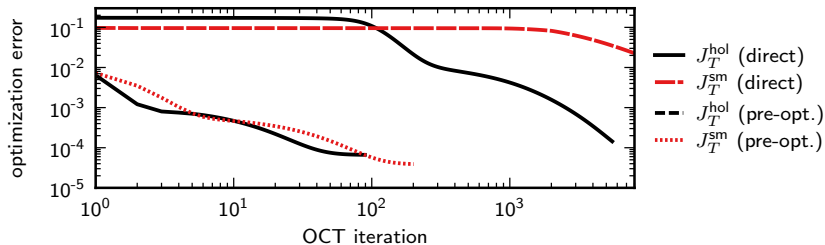
hybrid optimization schemes

combine gradient-free and gradient-based optimization in
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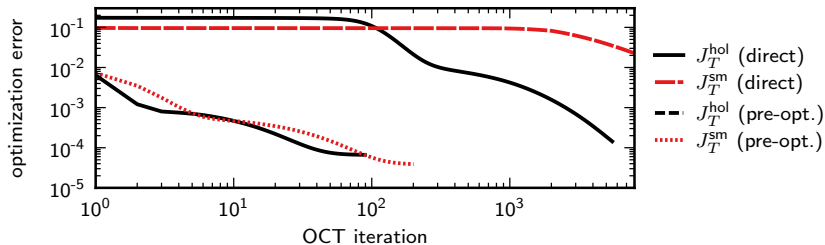


Goerz et al. EPJ Quantum Tech. 2, 21 (2015).

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Bridging the gap to experiment:
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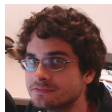
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Stanford



Kurt Jacobs
ARL



quantum dynamics and control
www.qnet-library.net



github.com/mabuchilab/QNET

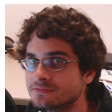
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github.com/mabuchilab/QNET

Thank you!