

Prospects of Optimal Control for Superconducting Circuits

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Army Research Lab

Yale Quantum Institute Seminar

January 14, 2020

optimum quantum control

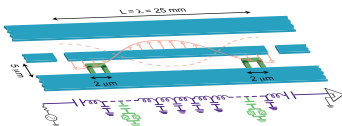
steer quantum system in some desired way

optimum quantum control

steer quantum system in some desired way

physically:

transmons with shared
transmission line:



Blais et al. PRA 75, 032329 (2007)

e.g. CNOT gate:

$$|00\rangle \rightarrow \text{CNOT } |00\rangle = |00\rangle$$

$$|01\rangle \rightarrow \text{CNOT } |01\rangle = |01\rangle$$

$$|10\rangle \rightarrow \text{CNOT } |10\rangle = |11\rangle$$

$$|11\rangle \rightarrow \text{CNOT } |11\rangle = |10\rangle$$

with *same* control $\epsilon(t)$.

optimum quantum control

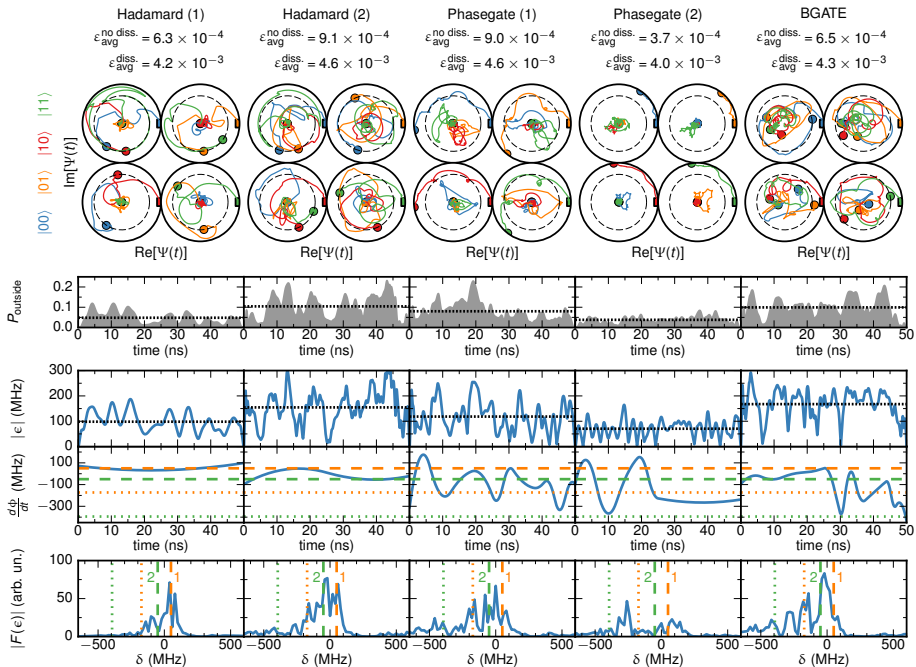
steer quantum system in some desired way

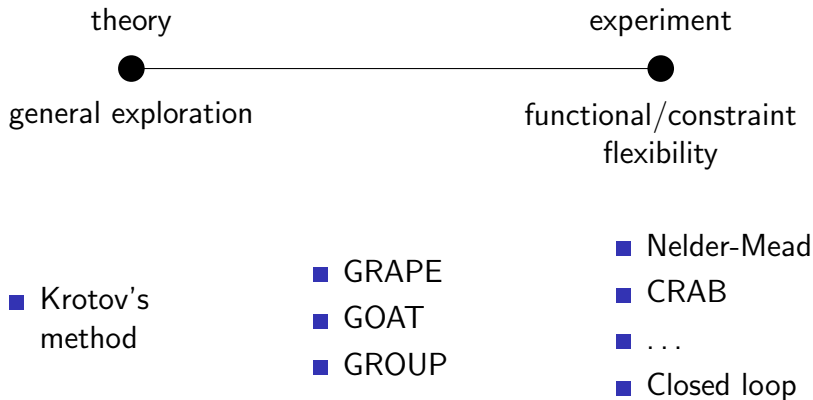
mathematically:

minimize functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$

$$= J_T(\{|\phi_k(T)\rangle\}) + \sum_l \int_0^T g_a(\epsilon_l(t)) dt$$

$$\text{e.g. } J_{T,\text{re}} = 1 - \frac{1}{N} \text{Re} \left[\sum_{k=1}^N \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle \right]$$





<https://github.com/qucontrol/krotov>

The screenshot shows a web browser window displaying the documentation for the Krotov package. The browser's address bar shows the URL `qucontrol.github.io`. The page has a dark blue header with the Krotov logo and version `1.0.0`. Below the header is a search bar labeled "Search docs". A dark sidebar on the left contains a "CONTENTS:" section with links to "Krotov Python Package", "Contributing", "Credits", "Features", "History", "Introduction", "Krotov's Method", "Using Krotov with QuTiP", "Examples", and "How-Tos". At the bottom of the sidebar is a "Doctr" button and a version selector set to "v1.0.0 (latest release)". The main content area has a heading "Docs » Welcome to the Krotov package's documentation!". Below this is a large heading "Welcome to the Krotov package's documentation!" with a link icon. A row of badges includes "github qucontrol/krotov", "docs doctr", "pypi v1.0.0", "chat", and "on gitter". Another row of badges shows "build passing", "build passing", "codecov 95%", and "License BSD". A third row includes "launch binder", "DOI 10.21468/SciPostPhys.7.6.080", and "Contents:". Under "Contents:", there is a list: "Krotov Python Package" (bullet point), "Purpose" (circle point), "Prerequisites" (circle point), "Installation" (circle point), and "Usage" (circle point).

Krotov
1.0.0

Search docs

CONTENTS:

- Krotov Python Package
- Contributing
- Credits
- Features
- History
- Introduction
- Krotov's Method
- Using Krotov with QuTiP
- Examples
- How-Tos

Doctr v1.0.0 (latest release)

Docs » Welcome to the Krotov package's documentation!

Welcome to the Krotov package's documentation!

github qucontrol/krotov docs doctr pypi v1.0.0 chat on gitter

build passing build passing codecov 95% License BSD

launch binder DOI 10.21468/SciPostPhys.7.6.080

Contents:

- Krotov Python Package
 - Purpose
 - Prerequisites
 - Installation
 - Usage

<https://github.com/qucontrol/krotov>

Optimization of a State-to-State Transfer in a Lambda System in the RWA

Optimization of a Dissipative State-to-State Transfer in a Lambda System

Optimization of Dissipative Qubit Reset

Optimization of an X-Gate for a Transmon Qubit

Optimization of a Dissipative Quantum Gate

Optimization towards a Perfect Entangler

Ensemble Optimization for Robust Pulses

Optimization with numpy Arrays

How-Tos

Other Optimization Methods

Related Software

Doctr v1.0.0 (latest release) ▾

Examples

render on [nbviewer](#) [launch](#) [binder](#)

- [Optimization of a State-to-State Transfer in a Two-Level System](#)
- [Optimization of a State-to-State Transfer in a Lambda System in the RWA](#)
- [Optimization of a Dissipative State-to-State Transfer in a Lambda System](#)
- [Optimization of Dissipative Qubit Reset](#)
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- [Ensemble Optimization for Robust Pulses](#)
- [Optimization with numpy Arrays](#)

Previous Next

SciPost Phys. 7, 080 (2019)

The screenshot shows a web browser window with the address bar displaying `scipost.org/10.21468/SciP`. The page features the SciPost logo in the top left and a search bar in the top right. Below the logo is a blue navigation bar with a hamburger menu icon. The breadcrumb trail reads: Journals / SciPost Physics / Vol. 7 Issue 6 / Krotov: A Python implementation of Krotov's method for quantum optimal control. The article title "Krotov: A Python implementation of Krotov's method for quantum optimal control" is displayed in a large blue font. Below the title, the authors are listed: Michael H. Goerz, Daniel Basilewitsch, Fernando Gago-Encinas, Matthias G. Krauss, Karl P. Horn, Daniel M. Reich, and Christiane P. Koch. The publication information states "SciPost Phys. 7, 080 (2019) · published 12 December 2019". At the bottom, there is a row of links: the DOI `doi: 10.21468/SciPostPhys.7.6.080`, buttons for "pdf", "BiBTeX", "RIS", and "Submissions/Reports", and a "Check for updates" button.

SciPost

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Journals / SciPost Physics / Vol. 7 Issue 6
/ Krotov: A Python implementation of Krotov's method for quantum optimal control

SciPost Physics

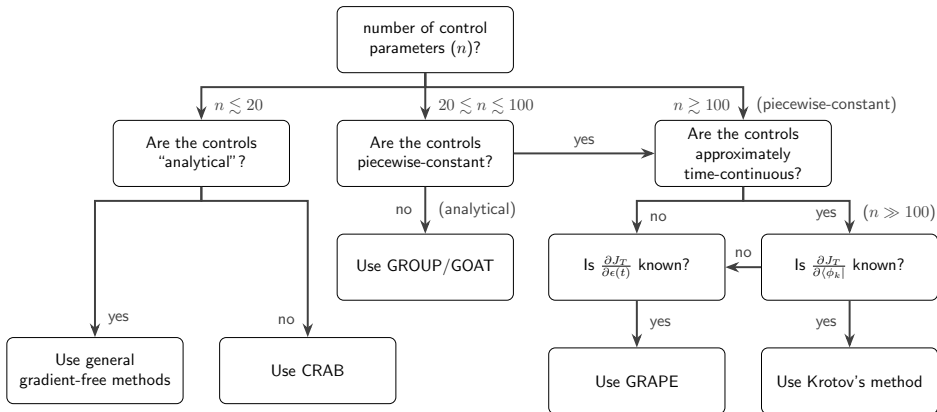
Home Authoring Refereeing Submit a manuscript About

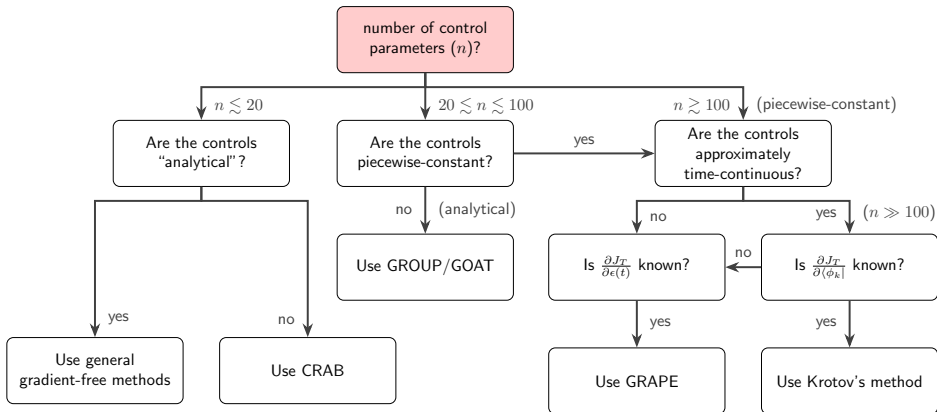
Krotov: A Python implementation of Krotov's method for quantum optimal control

Michael H. Goerz, Daniel Basilewitsch, Fernando Gago-Encinas, Matthias G. Krauss, Karl P. Horn, Daniel M. Reich, Christiane P. Koch

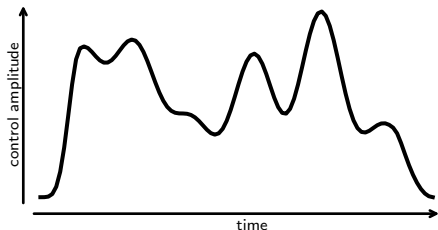
SciPost Phys. 7, 080 (2019) · published 12 December 2019

doi: 10.21468/SciPostPhys.7.6.080 pdf BiBTeX RIS Submissions/Reports Check for updates





functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_l(t)\}]$

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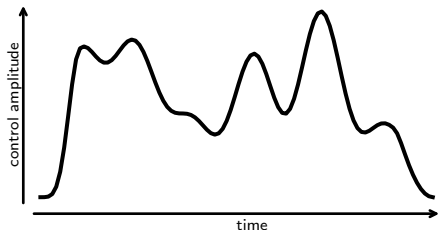
time-continuous fields



functional derivative $\frac{\delta J}{\delta \epsilon_I(t)}$

problem:
interdependence of field
and states

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



time-continuous fields



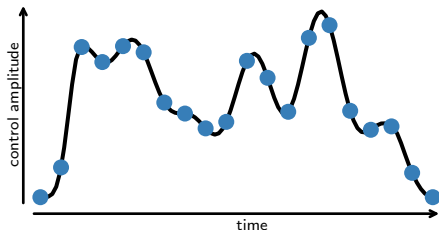
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Krotov's method:

- Add “smart zero” to functional to disentangle interdependence of states and field
- Necessary and sufficient conditions for monotonic convergence

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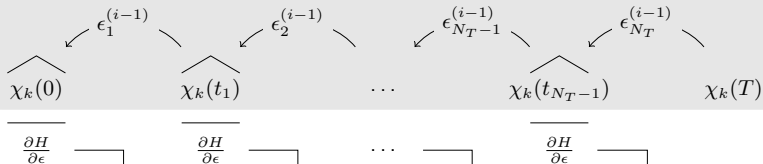
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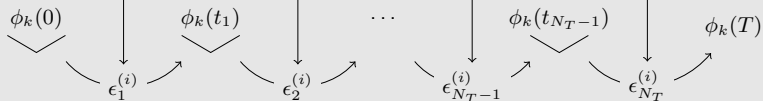
Krotov's method (discretized)

sequential scheme:

① backward-propagation and storage with guess control



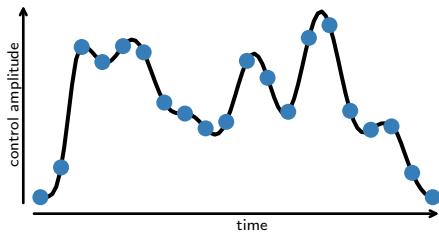
② forward-propagation with updated control



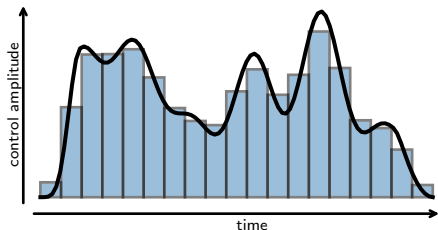
boundary condition:

$$|\chi_k(T)\rangle = -\frac{\partial J_T}{\partial \langle \phi_k(T) |}$$

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



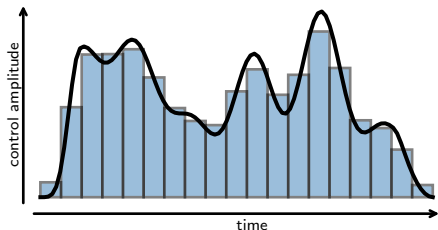
functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



piecewise constant fields

$\Downarrow$
derivative $\frac{\partial J}{\partial \epsilon_I(t_n)}$

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



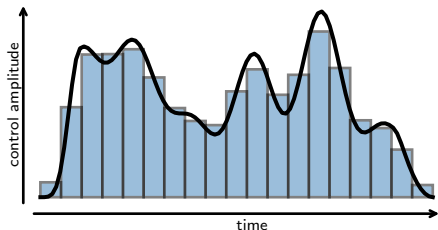
piecewise constant fields

$\Downarrow$
derivative $\frac{\partial J}{\partial \epsilon_I(t_n)}$

Gradient Ascent Pulse Engineering (GRAPE):

- Discretize first! Then calculate the gradient
- L-BFGS-B: Quasi-Hessian for faster convergence

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



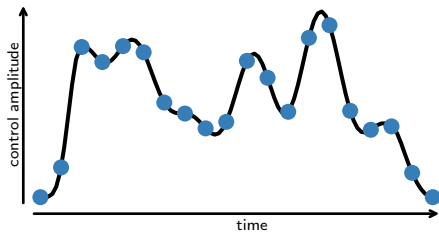
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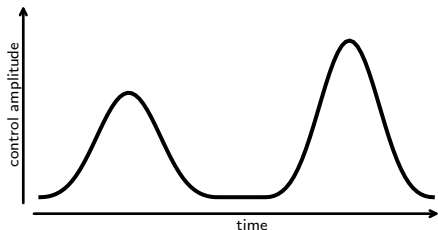
Gradient Ascent Pulse Engineering (GRAPE):

$$\begin{aligned} \frac{\partial \tau_k}{\partial \epsilon_n} &= \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle = \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_n \dots \hat{\mathbf{U}}_1 | \phi_k \rangle \\ &= \underbrace{\langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_{n+1} }_{\langle \chi_k(t_{n+1}) |} \frac{\partial \hat{\mathbf{U}}_n}{\partial \epsilon_n} \underbrace{\hat{\mathbf{U}}_{n-1} \dots \hat{\mathbf{U}}_1 | \phi_k \rangle}_{| \phi_k(t_n) \rangle} \end{aligned}$$

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$

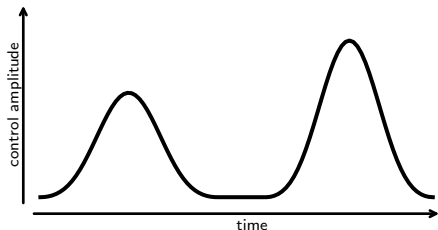


simple parametrized fields



gradient-free methods,
e.g. CRAB

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



simple parametrized fields

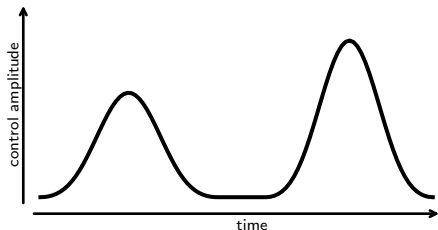


gradient-free methods,
e.g. CRAB

$$\epsilon(t) = E_1 G(t_1, \sigma_1, t) + E_2 G(t_2, \sigma_2, t)$$

$$\epsilon(t) = \sum_{i=1}^{10} (a_n \cos(\omega_n t) + b_n \sin(\omega_n t))$$

functional $J[\{|\phi_k(t)\rangle\}, \{\epsilon_I(t)\}]$



simple parametrized fields



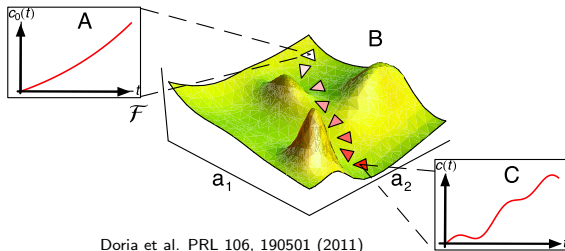
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... Filter function of control electronics – non-analytic!

gradient-free optimization



e.g. Nelder-Mead (simplex), genetic algorithms. . .

GROUP & GOAT: apply gradient to parametrized pulses

$$\epsilon(t) = S(t) \sum_n c_n f_n(t) \quad \text{or generally} \quad \epsilon(t) = \epsilon(\{c_n\})$$

$$\Rightarrow \frac{\partial J}{\partial c_n} = \sum_i \frac{\partial J}{\partial \epsilon_i} \frac{\partial \epsilon_i}{\partial c_n}$$

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GROUP (collection of methods): chain rule

Sørensen et al., Phys. Rev. A 98, 022119 (2018)

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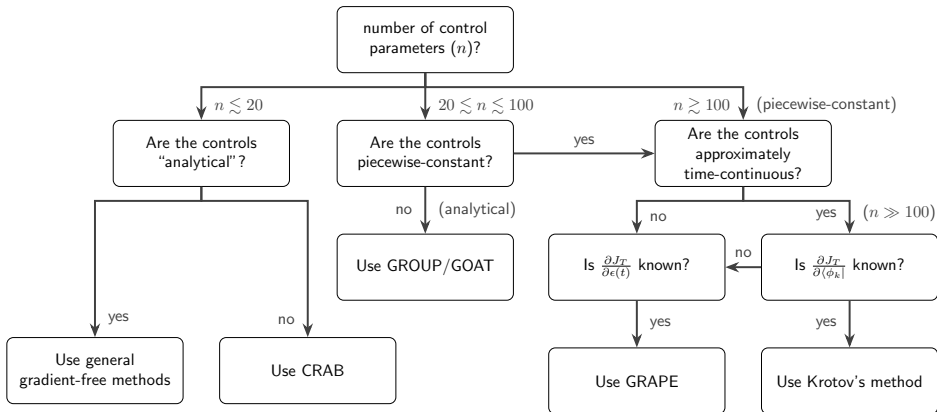
GROUP (collection of methods): chain rule

Sørensen et al., Phys. Rev. A 98, 022119 (2018)

GOAT: evaluate functional and gradient at the same time
(once per parameter)

Machnes et al., Phys. Rev. Lett. 120, 150401 (2018)

$$\partial_t \begin{pmatrix} U \\ \partial_{c_n} U \end{pmatrix} = -\frac{i}{\hbar} \begin{pmatrix} H & 0 \\ \partial_{c_n} H & H \end{pmatrix} \begin{pmatrix} U \\ \partial_{c_n} U \end{pmatrix}$$

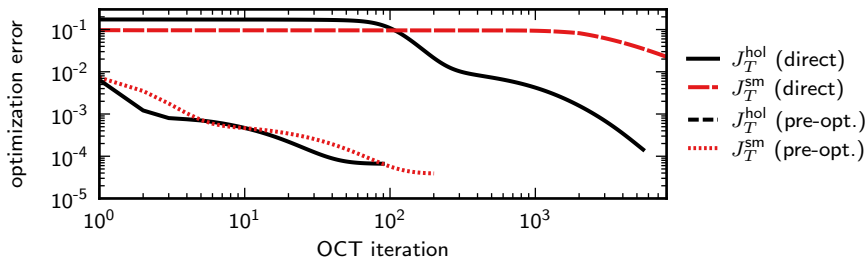


“hybrid” (multi-stage) methods (example: RIP gate)

- 1 Start with analytical formula, optimize free parameter with **simplex**
- 2 Use simplex-optimized control as starting point for **gradient-based** method

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- 1 Start with analytical formula, optimize free parameter with **simplex**
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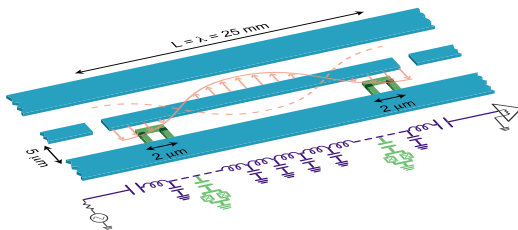


⇒ Goerz et al. EPJ Quantum Tech. 2, 21 (2015)

Charting the circuit QED design landscape using optimal control theory

Goerz et al. npj Quantum Information 3, 37 (2017)

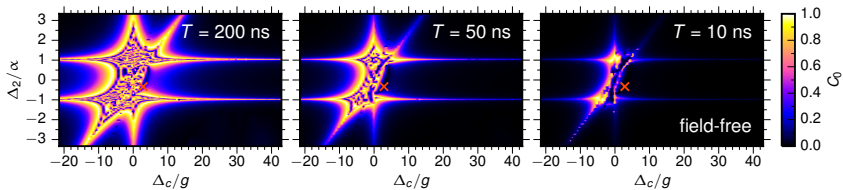
two transmons with shared transmission line bus:

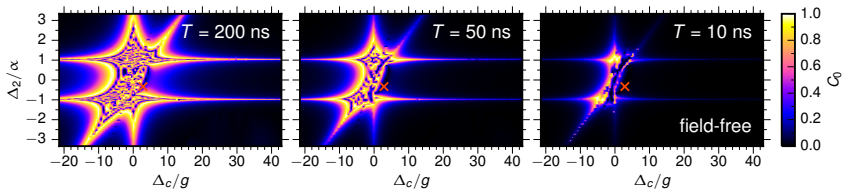


Blais et al. PRA 75, 032329 (2007)

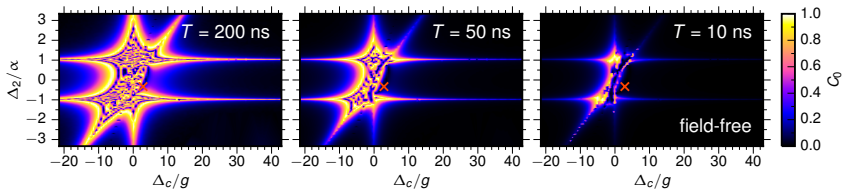
$$\begin{aligned} \hat{H} = & \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 + \frac{\alpha_1}{2} \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \frac{\alpha_2}{2} \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2 \\ & + g_1 (\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2 (\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger) + u^*(t) \hat{\mathbf{a}} + u(t) \hat{\mathbf{a}}^\dagger \end{aligned}$$

parameter landscape: $\Delta_c/g,$ Δ_2/α

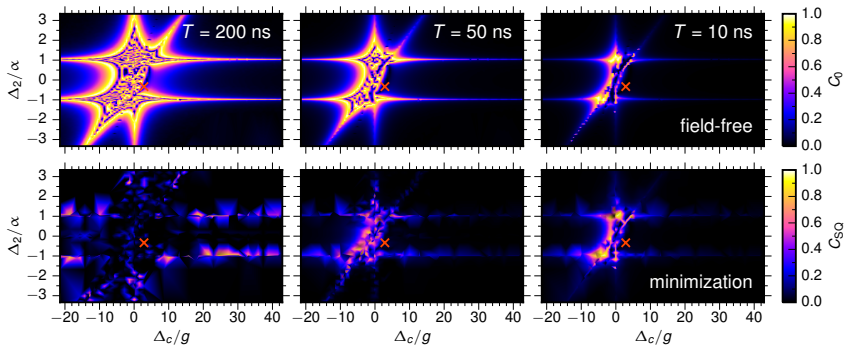


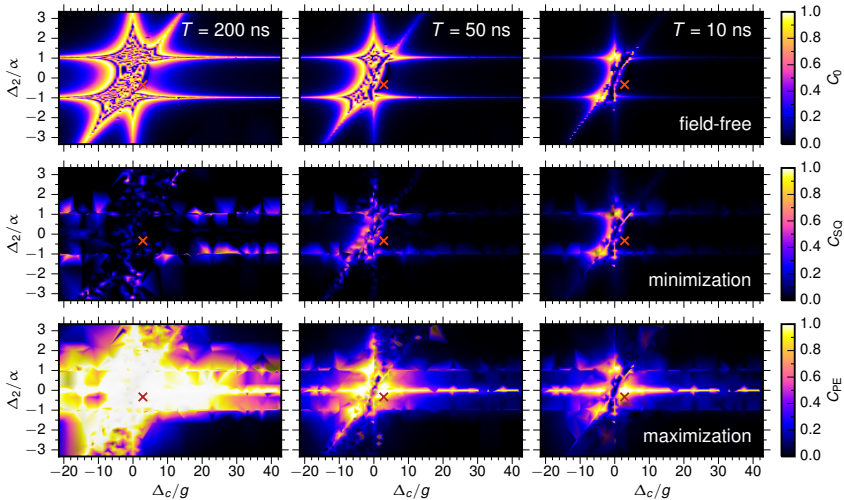


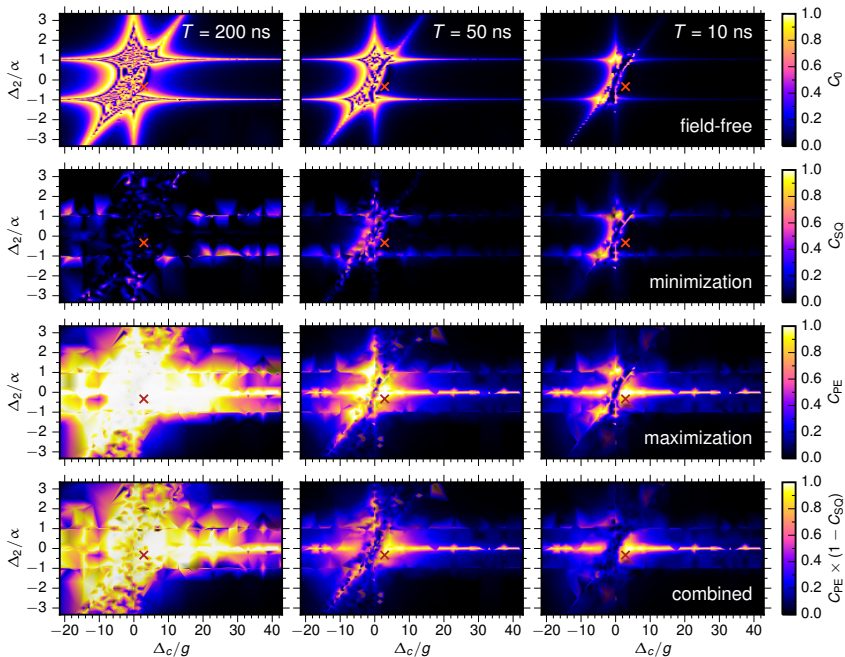
universal quantum computing:
perfect entangler *and* local gates

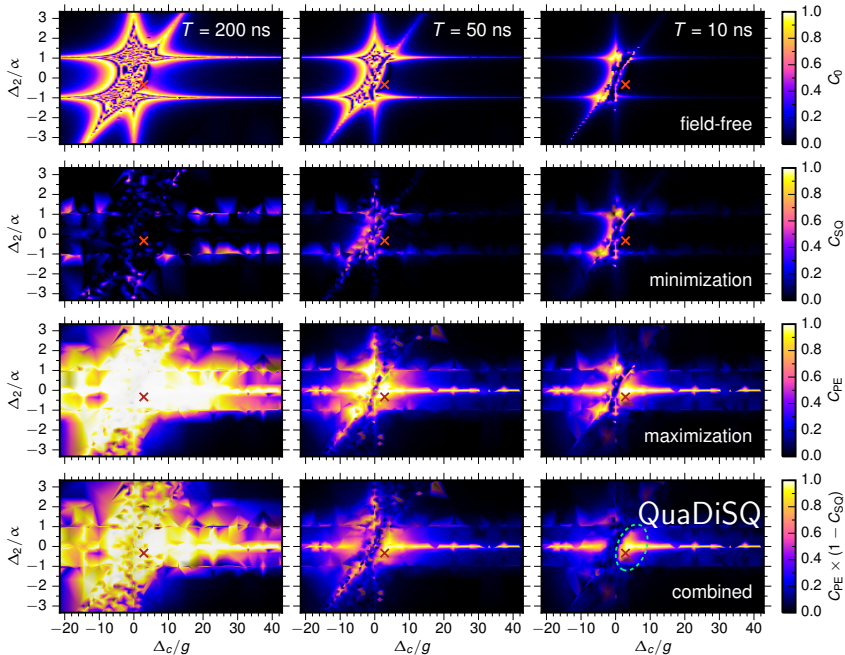


optimize for maximum / minimum entanglement







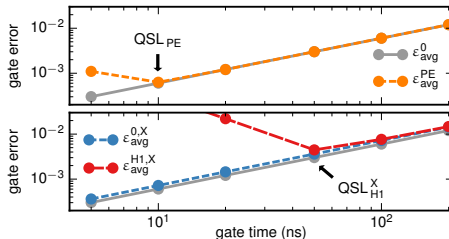


optimize for *specific* set of universal gates:

- Hadamard, Phase single qubit
- BGATE entangler

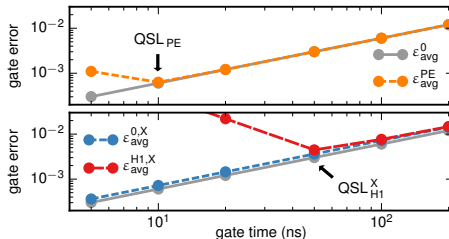
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optimize for *specific* set of universal gates:

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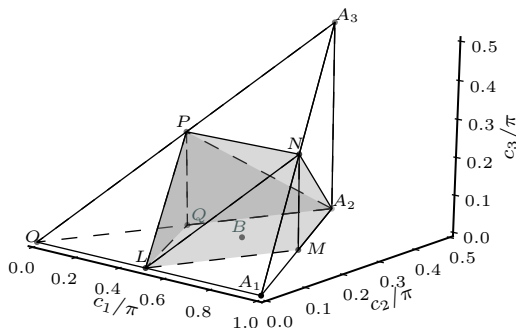
minimum gate duration: 50 ns

gate errors:

- 10⁻⁴ without dissipation
- 10⁻³ with dissipation

optimizing for entanglement

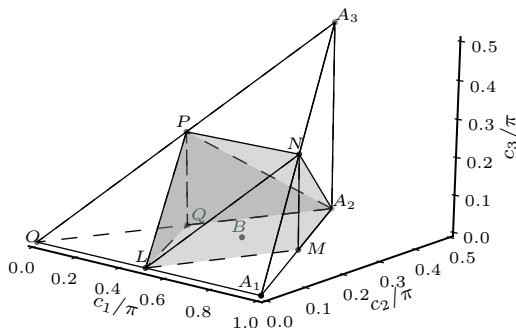
optimizing for entanglement



Cartan decomposition:

$$\hat{U} = \hat{k}_1 \exp \left[\frac{i}{2} (c_1 \hat{\sigma}_x \hat{\sigma}_x + c_2 \hat{\sigma}_y \hat{\sigma}_y + c_3 \hat{\sigma}_z \hat{\sigma}_z) \right] \hat{k}_2$$

optimizing for entanglement



Cartan decomposition:

$$\hat{U} = \hat{k}_1 \exp \left[\frac{i}{2} (c_1 \hat{\sigma}_x \hat{\sigma}_x + c_2 \hat{\sigma}_y \hat{\sigma}_y + c_3 \hat{\sigma}_z \hat{\sigma}_z) \right] \hat{k}_2$$

$\Rightarrow$ application: Goerz et al. Phys. Rev. A 91, 062307 (2015)

Outlook: learning from machine learning

Reinforcement learning for quantum control

- Use a neural net (AlphaZero)
to choose pulse amplitude in each time slice.

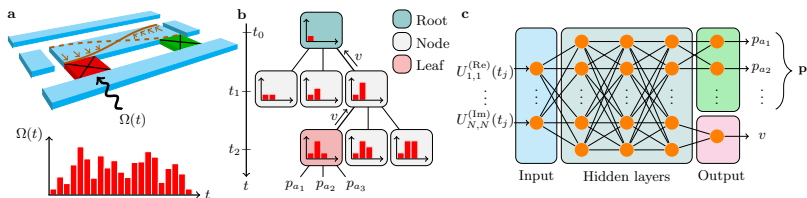


Fig 1 from: Dalgaard et. al, arXiv:1907.05672 (2019)

Automatic Differentiation

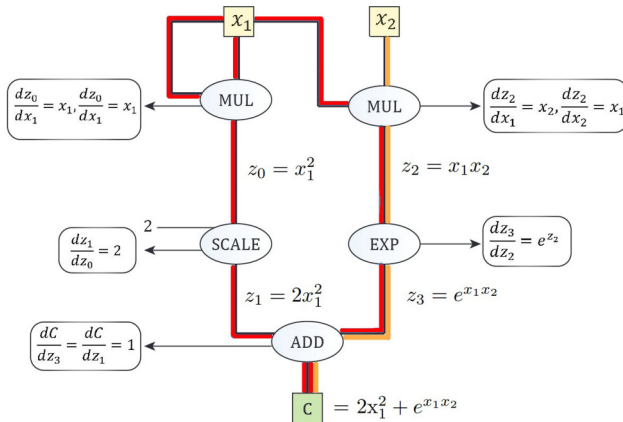


Fig 12 from: Abdelhalfez et. al, PRA 99, 052327 (2019)

Automatic differentiation for MCWF propagation & optimization functional using Google Tensorflow

PHYSICAL REVIEW A **95**, 042318 (2017)

Speedup for quantum optimal control from automatic differentiation based on graphics processing units

Nelson Leung,^{1,*} Mohamed Abdelhafez,¹ Jens Koch,² and David Schuster¹

¹The James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

²Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA

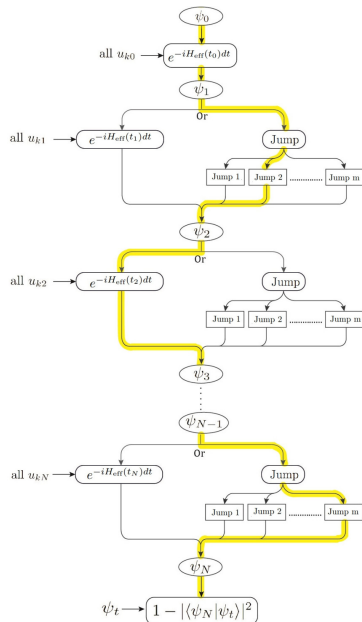
PHYSICAL REVIEW A **99**, 052327 (2019)

Gradient-based optimal control of open quantum systems using quantum trajectories and automatic differentiation

Mohamed Abdelhafez,^{1,*} David I. Schuster,¹ and Jens Koch²

¹The James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

²Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA



perfect entanglers functional:

$$\begin{aligned} F_{PE} = & \left(\frac{1}{\det U_B} \right) \left(\frac{1}{4} (\text{tr}^2[U_B^T U_B] - \text{tr}[U_B^T U_B U_B^T U_B]) \right) \left(\frac{1}{16} \text{Re}^2[\text{tr}[U_B^T U_B]] \right) + \\ & + \left(\frac{2}{\det U_B} \right) \left(\frac{1}{4} (\text{tr}^2[U_B^T U_B] - \text{tr}[U_B^T U_B U_B^T U_B]) \right) \left(\frac{1}{16} \text{Im}^2[\text{tr}[U_B^T U_B]] \right) \\ & \left(\frac{1}{16} \text{Re}[\text{tr}^2[U_B^T U_B]] \right) \end{aligned}$$

U_B : projection into logical subspace, in Bell basis

perfect entanglers functional:

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U_B : projection into logical subspace, in Bell basis

- control constraints
- noise and filter function of control electronics

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$$\frac{\partial \tau_k}{\partial \epsilon_n} = \frac{\partial}{\partial \epsilon_n} \langle \phi_k^{\text{tgt}} | \phi_k(T) \rangle = \underbrace{\langle \phi_k^{\text{tgt}} | \hat{\mathbf{U}}_{N_T} \dots \hat{\mathbf{U}}_{n+1}}_{\langle \chi_k(t_{n+1}) |} \frac{\partial \hat{\mathbf{U}}_n}{\partial \epsilon_n} \underbrace{\hat{\mathbf{U}}_{n-1} \dots \hat{\mathbf{U}}_1 | \phi_k}_{| \phi_k(t_n) \rangle}$$

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express functional in terms of scalar values τ_{ik}

$$F_{PE} = F_{PE}(\{\tau_{ik}\})$$

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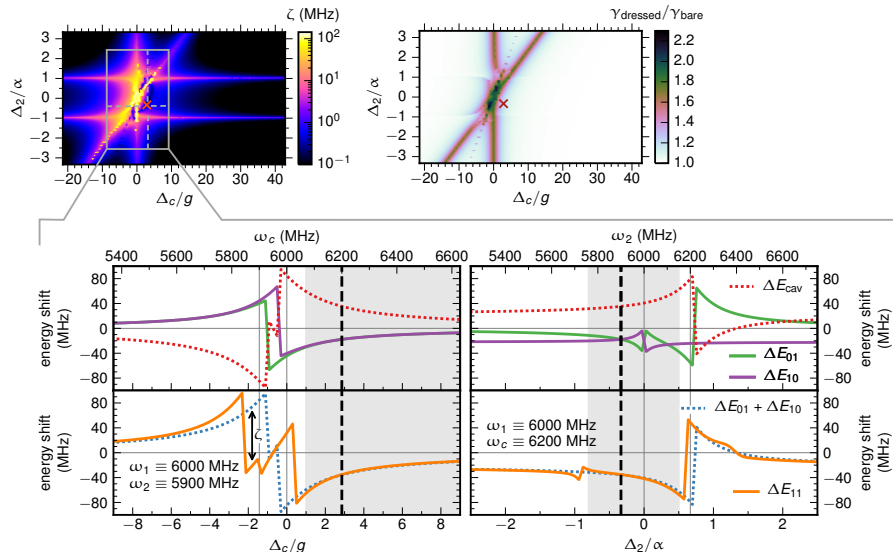
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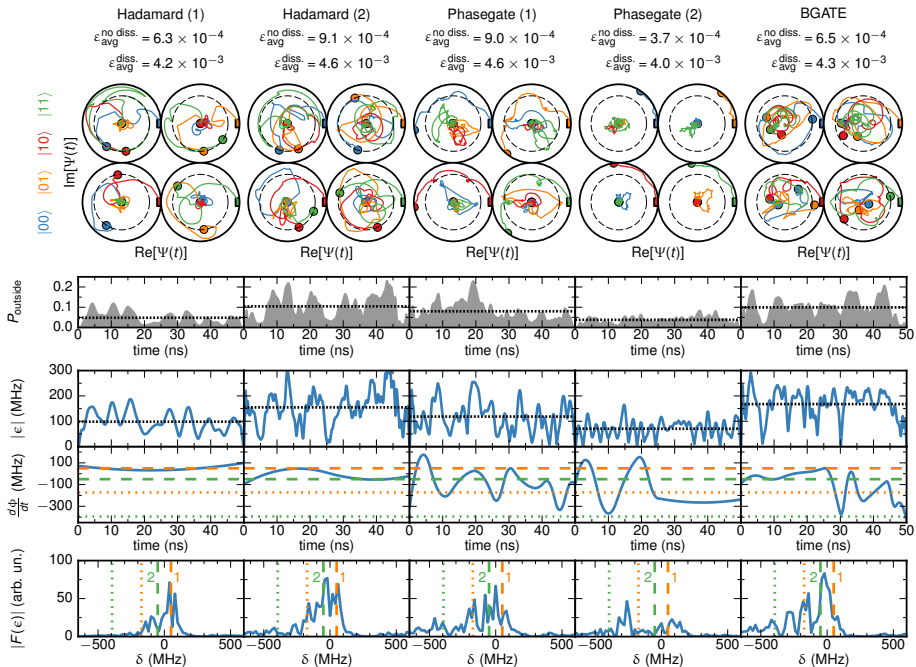
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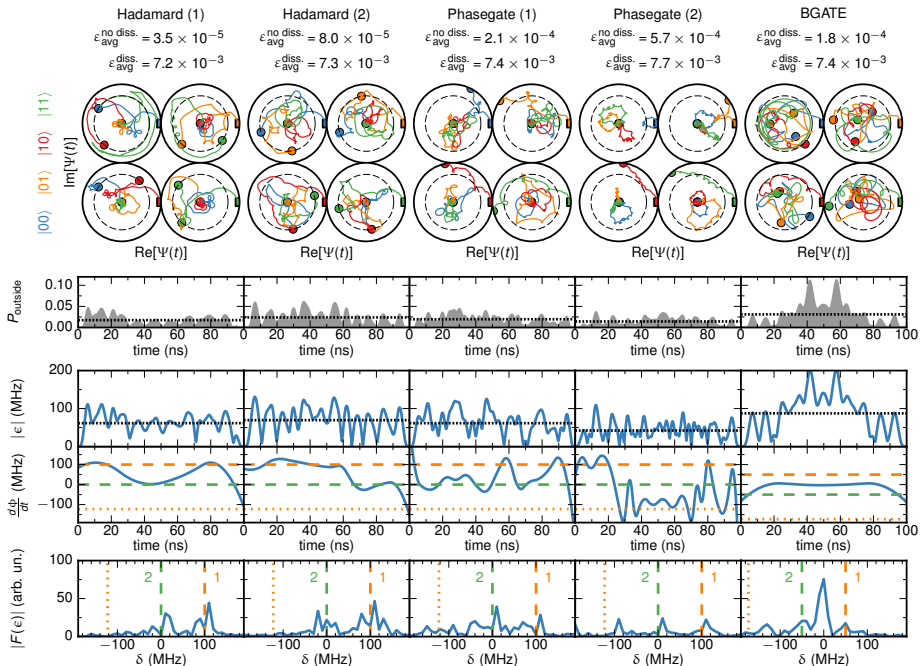
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Thank You

properties for field-free Hamiltonian:







obtained perfect entanglers in the Weyl chamber:

